THE
STUDY OF ELECTRICITY
BY THE DEDUCTIVE
METHOD

BY

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PREFACE

Many years ago the author discovered the fact that an endless flexible shaft revolving about its geometric axis was a helpful analogy of the transmission of energy by electricity. The applications of this analogy have been extended at intervals, and a pamphlet on the subject was published in 1915.

The analogy when used in connection with a single working hypothesis has universal application, affording the student the advantage of the deductive method of study. A considerable number of students have from time to time expressed their appreciation of the assistance they have derived from the use of this analogy.

Only quite recently has the attempt been made to extend the analogy and its accompanying working hypothesis to the solution of *all* the problems of electric transmission of energy by both direct and alternating current. The result has led to the publication of this book.

Caution—The student or the critical reader should keep clearly in mind that the analogy used as a basis for deductive study is not here presented as a *theory*. Neither the value nor the results of the method are at all dependent upon our views as to the probability of the real existence of the elementary flexible shafts referred to in the analogy. All teachers, I believe,

advocate and use analogies. What we claim for our analogy is its universal application in a given field—namely, transmission of energy by electricity.

I take pleasure in making here acknowledgment of the assistance of Mr. Harry B. Lindsay, who arranged much of the material in the book.

CONTENTS

INTRODUCTION

The various branches of modern science have been developed primarily and mainly by the use of the inductive method of study. When the deductive method has been applicable, it has usually resulted in a great saving of time to the student, and in added clearness of thinking. When the deductive method can be used, the student proceeds with confidence in the method, and an unusual degree of satisfaction in the results obtained. In the earlier studies in all branches of science, the approach was doubtless along the line of the inductive method. In some branches, like chemistry, there seems to be scarcely any other method possible. In modern astronomy, or in thermodynamics, on the contrary, the deductive method has wide application. It is clear that the deductive method requires some comprehensive law, principle, or analogy, as a basis for its application. Keppler discovered the laws of planetary motion mainly by the inductive method. By repeated and continuous observations made by himself, and by the study of what others had observed and plotted, he was finally able to enunciate the laws governing the motion of the planets, which are known as Keppler's Laws. Later, Sir Isaac Newton discovered the law of gravitation. With this law as a basis, the mathematician is easily able to deduce the laws of Keppler. The inductive process was the work of a lifetime. By the deductive

process, the mathematician can prove the laws in a day. Astronomy early took a high rank among the sciences. It held this rank because it was able to predict results. The astronomer not only explains why an eclipse occurs, but he predicts by the deductive method the exact time of its occurrence.

In some, if not all the subjects which have invited scientific investigation, there have been at times erroneous or inadequate reasons put forward to explain certain phenomena. Heat was at one time thought to be a material substance called caloric. The term latent heat, when first used, was applied to the unexpected appearance of heat, the cause of which was not really known, such, for instance, as the heat developed by friction. This heat was said to have been latent in a body until called forth by some peculiar conditions; but the relations between the conditions which resulted in the heat, and the effect produced, were not understood. When it was discovered that heat was a mode of molecular motion, the term latent heat was retained, but given an entirely new meaning.

In each of the branches of science, there has come a time when the discovery of a new truth has marked an epoch of change, and of important progress in the subject. In chemistry this epoch came with the recognition of the indestructibility of matter. So long as it was thought that material might be destroyed in the process of an experiment, little value could be attached to the results. The acceptance of the idea of the indestructibility of energy, or the discovery of

what is known as the conservation of energy, gave
us the science of thermodynamics, and the rapid
development in the use of steam and in the application
of its energy to the commercial uses of modern
civilization.

When we come to the subject of electricity, we find
that while it long occupied a place in the text books
of physics, it had little or no commercial value until
the invention of the dynamo, or electric generator, in
about 1876. When it was found that mechanical
energy delivered to a generator could be transmitted
with little difficulty through long distances, the com-
mercial value of this mode of transmitting energy
brought the subject of electricity into prominence.
In the case of electricity, the inductive method has
been entirely successful in discovering all its functions
and uses, and in giving to the world the full benefit
of its great possibilities.

In preparing this volume from the standpoint of a
new approach to the study of electricity, I have had
in mind, primarily, the advantages of the logical
processes of thought, the economy of time, and inci-
dentally, the financial as well as the educational value
of the deductive method of study, to the *student* and
the *engineer*.

I have no doubt that teachers as well as students
have felt the lack of some more definite guiding prin-
ciple relating to the various phenomena of that seem-
ingly elusive and mysterious thing which we call
electricity. Long experience in the use of electrical
machinery, and in the handling of electric power,

brings with it a certain confidence as to what to do under certain circumstances; what various mechanisms can accomplish, and how they should be used. But this confidence is much more readily acquired when the experience is preceded by familiarity with the method of study disclosed in this volume.

The principles and laws of magnetism and magnetic circuits have been assumed to be acceptable to the student in the form in which they are stated in the usual text book. Chapter I is a résumé and discussion of the essential facts of magnetism, stated as briefly as is consistent with the necessity for a clear understanding of the subject on the part of the student who is about to take up the study of electricity.

Attention is directed to the Appendix, wherein will be found discussion of points which, for the sake of clearness, have been inserted without comment in the text.

THE STUDENT'S GUIDE TO DEDUCTIVE THOUGHT

As a help to the student, there is placed at the beginning of each chapter a brief paragraph in which an attempt is made to indicate the train of thought which he will be asked to follow in that chapter.

CHAPTER I

STUDENT'S GUIDE

In this chapter will be found the elementary facts about magnets and magnetism. Magnetism is closely associated with all electrical phenomena, and it is therefore of the greatest importance to study this relation. Simple illustrations have been chosen for discussion to demonstrate the fundamental principles involved.

MAGNETISM

Before proceeding to the study of electricity, it is essential that the elementary facts of magnetism be reviewed. A magnet may exist for ages without the appearance of electricity, but no energy is transmitted or applied by electrical means without the appearance of, and usually the assistance of magnetism.

It was early observed that a mineral known as magnetite (Fe_3O_4) had, in its natural state, the property of attracting to itself and holding pieces of iron and steel. This property of attraction is called magnetism, and the body possessing it is called a magnet. A piece of iron ore which possesses inherent magnetism is called a natural magnet. A piece of steel

which has been magnetized is called an artificial
magnet.   There are other substances capable of
being attracted by a magnet, but their susceptibility
to magnetism is small and relatively of little import-
ance; cobalt, nickel, chromium, and manganese are
such materials.

The most common example of a magnet is the
ordinary woodsman's or mariner's compass needle,
which is simply a small magnet pivoted and mounted
so that when undisturbed by local magnetic influences
it will indicate magnetic north and south.   That end
of the needle which always points north is called the
north pole of the magnet, and the other end is called
the south pole.   This action of the compass needle is
due to the fact that the earth itself exhibits magnetic
polarity, there being a magnetic pole in the vicinity of
each of the geographic poles.

To explain the influence of one magnet on another,
or of a magnet on a piece of steel, the existence of lines
of magnetic force has been assumed; and the region
about a magnet where magnetic effects may be de-
tected is called a field of force or magnetic field, and
is assumed to be traversed by a greater or lesser num-
ber of lines of force according to the strength of the
magnet.   (See Fig. 1.)   It will be seen a little later
that magnetic lines of force always seem to exist in the
presence of an electric current, the magnetic field thus
produced being similar in properties to that existing
around a magnet.

It is convenient for discussion, and necessary for the
explanation of certain electrical phenomena, to state

a few of the conventions commonly applied to magnetic lines and fields of force.

Lines of force (sometimes called magnetic flux) emerge from north magnetic poles and enter south poles.

Lines of force are closed loops, tending always to become shorter, having a repellant effect on each other and being established with much greater ease in steel and iron than in any other substance.
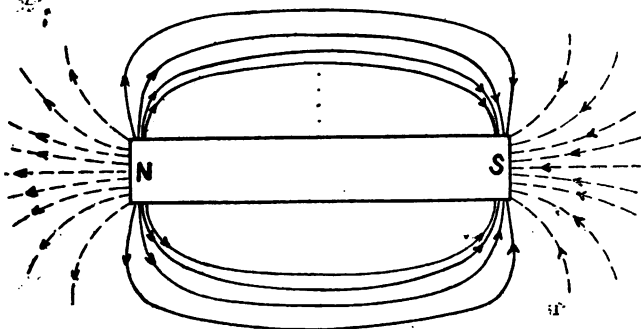
FIG. 1

The repellant effect is assumed to explain the noticeable repulsion of like poles; and the tendency to become shorter, combined with the ease of the establishment of lines in iron and steel, is an assumption made in order to explain the attractive force exerted by a magnet on a bit of iron or steel.

The property of iron and steel which promotes the apparent strength of lines of force is called permeability; thus of two bars of iron subjected to the same magnetizing force, the one which appears to sustain the greater number of lines of force is said to have the

higher permeability; or expressed negatively, to have
the lower reluctance. In general, the softer the iron,
the higher the permeability.

Iron and steel differ also to a great degree in reten-
tiveness of magnetism; soft iron loses nearly all of its
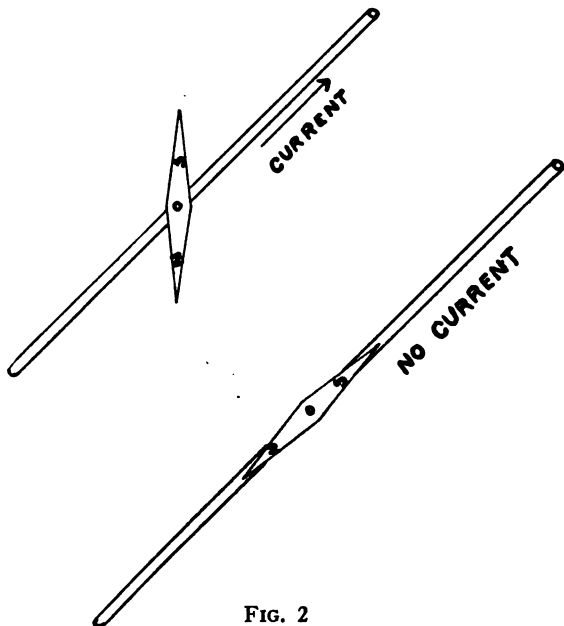
FIG. 2

magnetism when the magnetizing force is removed,
whereas steel retains a considerable proportion of its
magnetism indefinitely.

The magnetism evidenced whenever an electric
current is established, and which persists while the
current is flowing, but disappears when the current
ceases to flow, is called electro-magnetism.

To be assured of the presence of a magnetic field in the space about a conductor carrying an electric current, we have only to pass a compass needle over or under the conductor. (See Fig. 2.) The violent

POSITIVE
CURRENT.

NEGATIVE
CURRENT.
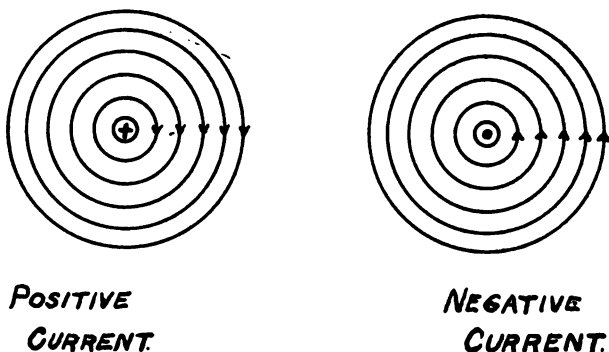
FIG. 3
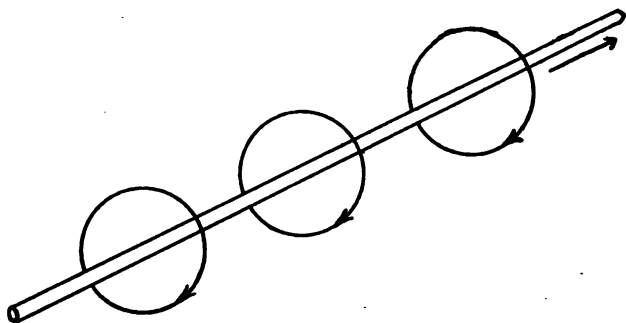
FIG. 4

deflection of the needle is visible evidence of the action of a force, and simple experiments with the compass needle and a straight conductor carrying a current have led to the following common assumptions:

The lines of force are circles concentric with the conductor (Fig. 3). Their direction is clockwise when viewed so that the current is in the positive direction (Figs. 3 and 4).

Quantitative measurements show that in air or in any non-magnetic material the distance from the wire at which force may be detected is directly proportional to the strength of the current.

If a wire be wound into a coil, the lines of force produced by one turn act in the same direction as those of the adjoining turns, and in this manner a field of force of considerable strength may be established by a very moderate current. By this means magnets called electro-magnets may be made, which are dependent for magnetism upon the current in the coils of wire with which they are wound. When arranged in proper mechanical form, a magnet may be attached to great loads of metal simply by passing current through the magnet coils, and the release of the material lifted is accomplished by stopping the flow of current.

Electro-magnets form the actuating force of a great number of electrical devices, such as telegraph sounders, annunciators, door openers, relays, circuit-breakers, watchman clocks, and electric devices of many sorts.

It is customary to speak of the path of the lines of force as the magnetic circuit; and where this path is in iron or steel throughout most of its length, no attention is given to the space outside the metal which constitutes the core of the magnet, and calculations

are made as if all of the magnetic flux traversed the core.

Figure 5 shows the magnetic circuit of an ordinary electric bell. *A*, *B*, *C*, and *D* are made of soft iron. The coils of wire are slipped on over the magnet cores
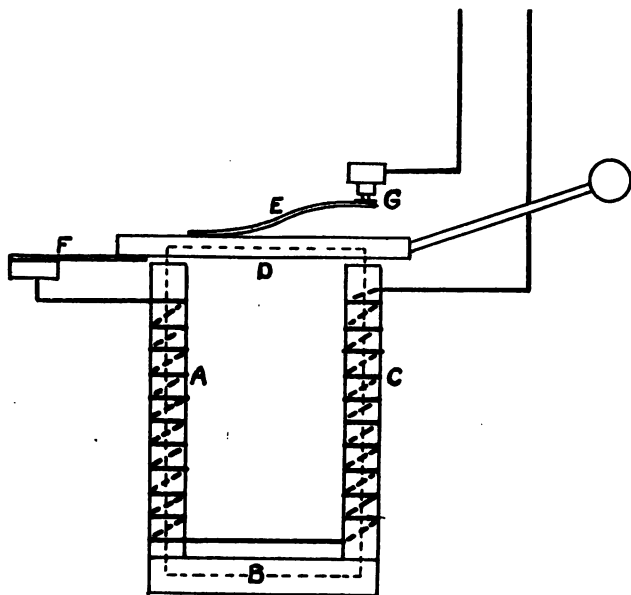
FIG. 5

A and *C* before the armature *D* is put in place. *F* is a spring support for the armature, which allows it to vibrate; *E* is a spring contact through which current is led to supply the magnet coils. The dotted line shows the path of the magnetic lines. The operation of the bell is as follows: current flows through the wire wound around the magnet cores *C* and *A*, through

FIG. 6

YOKE

POLE SHOE

POLE

ARMATURE

BASE
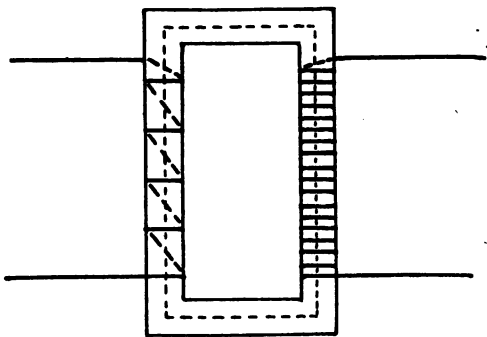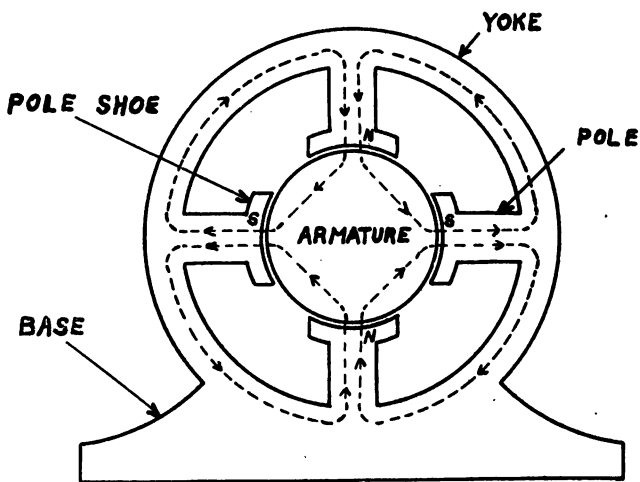
FIG. 7

the spring F, armature D, and out at E. The magnetic attraction of A and C for D causes the armature to draw E away from the fixed contact G, breaking the current. As soon as the current is broken, the magnetism is destroyed and spring F returns D to the position shown in Fig. 5, when the action is repeated, giving a rapid vibratory motion to the armature which carries the bell tapper.

Figures 6 and 7 show the magnetic circuits of a transformer and a four-pole generator respectively. The dotted lines represent the path of the lines of magnetic force. These pieces of apparatus are discussed in later chapters with respect to their electrical characteristics, and are introduced here simply to show the magnetic circuits of typical cases.

CHAPTER II

STUDENT'S GUIDE

In this chapter the fundamental mechanical quantities are stated and defined. These are the basic definitions which we must have well in mind before we can proceed to the study of electrical transmission of energy with confidence in our ability to apply logical tests to the conclusions at which we wish to arrive.

MECHANICS

In order that we may profitably approach the subject of electrical transmission of energy, we must first review the principles of mechanics, stating definitions and recording certain formulæ for future reference.

DEFINITIONS

Mass is the quantity of matter of which a body is composed.

Motion is a change of position of a body relative to some point of reference.

Force is that which produces, tends to produce, or to modify motion.

Weight. Weight is the force exerted on a body by virtue of the attraction of gravity.

Velocity or Speed is the rate of motion or rate of change of position. Velocity is measured in feet per second, miles per hour, revolutions per minute, etc.

Acceleration is the rate of change of velocity. It is measured in feet per second per second, miles per hour per minute, degrees per second per second, etc. The application of force to a mass whose motion is restricted only by its inertia causes acceleration, and the product of the mass times the acceleration equals the force exerted. $F = MA$.

Angular Velocity. When a body rotates, its particles move in circles about some line in the body called the axis of rotation. The rate of motion of a particle along the circumference of one of these circles, expressed in degrees per unit of time, is called angular velocity. Angular velocity is numerically equal to the linear velocity of a point at unit radius from the axis of rotation.

Energy, Work, Power. Work is done when a force causes motion, and is equal to the product of the force and the distance through which it acts. Energy is capacity for doing work. Power is the rate of application of energy, or the rate of doing work. Energy and power are frequently confused; they are related thus, $P = \dfrac{E}{T}$, where P = power, E = energy, and T = time. One horsepower = 33,000 foot pounds per minute = 746 watts.

Stress and Strain. When force is applied to a body, whether motion results or not, the body is under stress; the resulting deformation is called strain.

Stress which tends to lengthen the object to which it is applied is called tension; stress which tends to shorten or squeeze the object is called compression; stress which tends to cut an object by moving one section across another is called shear; and stress which tends to twist an object is called torsion.

Rigidity, Flexibility. A body which resists deformation is said to be rigid, or to possess rigidity; a body which is easily bent is called flexible. It is to be noted that a body may be rigid with regard to one sort of stress, and flexible to another; for example, a flat piece of steel is rigid under tension, but flexible in bending.

Elasticity. A body which when strained returns completely or nearly to its unstressed dimensions, is said to be elastic. Most materials have a definite stress beyond which they lose their elasticity. When this elastic limit is reached, the material loses much of its rigidity. A non-elastic body is called *plastic.* Metals become plastic when strained beyond their elastic limit.

Torque. The torque or moment of a force about any axis is equal to the product of the force and its perpendicular distance from the axis.

Inertia. Inertia is that property of matter by virtue of which it resists change in its state of rest or motion.

The Conservation of Energy. An understanding of the principle of the conservation of energy is most important. Logic and experience indicate that the quantity of energy in the universe is constant, and the

energy which can be transferred from a body or a system of bodies is limited. Energy expended as work, is transformed into a different form; no energy is destroyed or created.

Power Transmitted by a Rotating Shaft. If a force P is exerted at a radius R from the axis of rotation of a shaft which makes N revolutions per minute, the torque exerted is PR, the distance traveled by the force is $2\pi RN$ per minute. The work done per revolution is $2\pi RP$, and per minute is $2\pi RNP$. $2\pi N$ is the angular velocity of the shaft in degrees per minute ($\pi = 180°$, in this sense), and PR is torque.

$2\pi RNP = (PR)(2\pi N) = MA =$ power, where $A = 2\pi N =$ angular velocity, and $M = PR =$ moment or torque.

This last expression should be thoroughly understood and memorized.

Power and energy should be as distinctly separate in the student's mind as speed and distance. For example, if a man walk at the rate of 4 miles per hour for 3 hours, he will travel 12 miles. Rate $\times$ time $=$ distance. If a man is buying energy at the rate of 75 horsepower and he uses energy at that rate for 8 hours, he has used 600 horsepower hours; or since one horsepower is equal to 33,000 foot pounds per minute, $75 \times 33,000 =$ the power in foot pounds per minute. Eight hours equal 480 minutes; therefore, $75 \times 33,000 \times 480 = 1,188,000,000$ foot pounds of energy used. Here, as before, power $\times$ time $=$ energy.

CHAPTER III

Electricity Defined. The Analogy Stated—The electric circuit; statement of working hypothesis. Algebraic Expression for Rate of Transmission of Energy. Deduction of Ohm's Law. Current Direction Defined —Alternating current. Analogy Applied to Particular Circuit. Electric Batteries—Primary cells; storage cells.

STUDENT'S GUIDE

Having covered the preliminary ground, we shall now proceed to the statement of the analogy on which the work is based. Then, with the outlining of a working hypothesis, we are equipped for the investigation of any electrical phenomenon. The first subject treated is Ohm's Law, after which a simple electric circuit is discussed in terms of the analogy. Electric batteries are described, with notes on the charging of storage cells.

The function of electricity is to transmit energy. This, so far as we are aware, is its only function. The energy is transmitted by means of a conductor, commonly a copper wire. In practice, energy is transmitted in this way several hundred miles with success, and with comparatively small losses due to transmission. We define electricity from its single function, and we say that, for our purpose, electricity is a method of transmitting energy. We need go no farther than this for a definition. We need not

assume that there is any such entity as electricity. It is just a method of doing something. We send energy a long distance by electricity, and we send parcels a long distance by express. The question is naturally raised, "But how is the energy transmitted? If we do not know exactly how it is transmitted, is the transmission like anything we do know about?"

Our answer to this question is in the affirmative, and we answer it by saying that THE TRANSMIS-SION OF ENERGY BY ELECTRICITY IS LIKE THE TRANSMISSION OF ENERGY BY AN ENDLESS SHAFT OF NEGLIGIBLE MASS, PERFECTLY FLEXIBLE WITH REGARD TO BENDING, BUT REASONABLY RIGID AND ELASTIC WITH REGARD TO TORSION AND REVOLVING ABOUT ITS GEOMETRIC AXIS. This is our comprehensive analogy for the trans-mission of energy by electricity. Following out this analogy, if we inquire what is the formula for the transmission of energy by shafting, the answer, known to every student familiar with the rudiments of mechanics, is this: "The rate of transmission of energy by shafting is the moment or torque on the shaft, multiplied by its angular velocity of rotation."

Now we may picture in our minds a conductor of electricity as composed of an indefinite number of infinitesimal flexible shafts, each capable of rotation about its own geometric axis. We may consider each of these elementary flexible shafts as made up of a line of molecules of the conductor. A good con-ductor is one whose molecules are of such shape and

character as to enable these elementary shafts to re-volve without undue interference with one another. A poor conductor of electricity may be considered as a body whose molecules are of such shape and char-acter, or have such relations to one another, that these lines of molecules mutually interfere and thus present resistance to rotation.

An electric circuit consists of a continuous con-ductor which is capable of sustaining an electric current. The circuit is always a closed curve, and ordinarily includes the following: a source of elec-tromotive force, a load, and a wire extending from one terminal of the source to the point of applica-tion of the energy (at the load) and back to the other terminal. Obviously, the source of the e.m.f. and the load itself are parts of the continuous path which constitutes the circuit. An "open circuit" occurs when the conductor is cut, or when a por-tion of it is replaced by a non-conductor; when the circuit is complete, it is referred to as a "closed circuit." It should be observed that the presence of current is always proof that a closed circuit exists, but that a circuit may be closed without the inclusion of a source of electromotive force, in which case no current will be present.

While it is unnecessary, as it is impossible, to answer all the questions that may be asked in con-nection with our analogy, such as how the molecules may connect themselves with one another, etc., it is helpful for the application of this analogy to the complicated problems arising in connection with

alternating currents, to inquire what we may consider as taking place in the magnetic field of the electric generator. We know that when a coil of copper wire is moved in the magnetic field, a current is set up in the coil. Here we supplement our analogy by a working hypothesis as to the action which takes place in the coil under these circumstances. This working hypothesis is as follows:

WHEN A CONDUCTOR IN A CLOSED CIR-CUIT IS MOVED IN A MAGNETIC FIELD, THE MOTION TENDS TO CONSTANTLY MAGNETIZE THE MOLECULES OF THE CONDUCTOR, THE INSTANTANEOUS POLAR AXES OF THE MOLECULES BEING AT RIGHT ANGLES TO THE LINES OF FORCE IN THE MAGNETIC FIELD, AND THE NORTH POLES IN THE GENERAL DIRECTION OF THE MO-TION OF THE CONDUCTOR RELATIVELY TO THE LINES OF FORCE IN THE FIELD.

We find that, following this working hypothesis in connection with our analogy, we are led to results that explain and agree with all of the principal phenomena connected with the electrical transmission of energy, either by direct or alternating current, including induction, self-induction, lag, lead, capacity, etc. We therefore present the application of this working hypothesis in some detail.

If the transmission of energy by electricity is like transmission by innumerable tiny flexible shafts revolving at high velocity, we can readily write the

algebraic expression for the rate at which energy is thus being transmitted.

Let A = the common angular velocity of the flexible shafts.

Let M = the combined torque on all the shafts.

Then it is well known and easily demonstrated that the rate at which energy (W) is being transmitted is

Rate of transmission = $A \times M$ Equation 1

Ohm's Law is expressed by the formula

$$E = C \times R \qquad \text{Equation 2}$$

in which

E = electromotive force (volts);

C = current (amperes);

and

R = resistance of the conductor.

Multiplying both members of Equation 2 by C gives

$$CE = C^2R \qquad \text{Equation 3}$$

But

CE = rate of transmission by electric current;

and

AM = rate of transmission by flexible shafts.

The two expressions CE and AM are identical in form, and each represents rate at which energy is being transmitted. Therefore, C in Equation 3 must be angular velocity, and E must be torque. Thus our analogy leads us to the correct expression for rate of electric transmission and also enables us to define *electromotive force as torque, and current as angular velocity*.

Making C = 1 in Equation 3, we have $E = R$.

Thus, R is the e.m.f. when C is unity. E is also the energy when $C = 1$; $(E \times 1 = E$ units of energy).

The power equation, $P = CE$, lends itself readily to deductive analysis, and mechanical considerations lead from it to the deduction of Ohm's Law.

If we take a simple closed circuit, in which a current is flowing, and examine it to determine the factors upon which the rate of application of energy depends, we observe that a certain resistance to rotation is
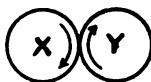
FIG. 8

encountered and overcome between the molecular filaments, the energy in this case being dissipated as heat.

This resistance is developed between bodies rotating in the same direction causing resistance at their lines of contact. In Fig. 8, X and Y represent cross-sections of molecular filaments of a body in which current is flowing. The doubling of the angular velocity of either filament will double the power required to overcome the resistance, and the doubling of the velocity of both will quadruple the power.

Hence the power required to overcome resistance is proportional to the square of the angular velocity of rotation of the filaments, and the general form of the power equation (see Chapter III) becomes

$$\text{Power} = MA = A^2K$$

or in electrical units

$$EC = C^2R$$

where R is a constant depending on the material and size of the conductor.

By dividing both sides of this expression by C we get $E = CR$, which is the familiar form of Ohm's Law.

In view of the foregoing, the so called "flow" of an electric current becomes an easily understandable mechanical operation involving a simple form of motion—namely, rotation. The direction of current flow is a mere convention, and we may choose one direction of rotation of our molecular filaments as
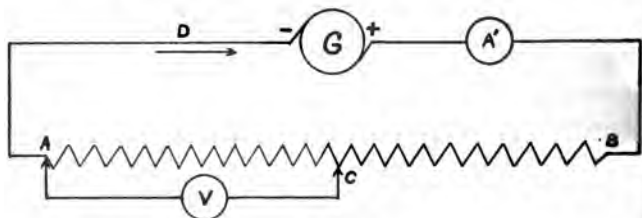
FIG. 9

corresponding to the positive direction of current flow, and the reverse for negative current. If the applied torque is oscillating, then the motion accompanying the electric transmission of energy will be oscillating rotary motion, and the current is said to be alternating.

As stated above, the direction of rotation of the molecular filaments could be chosen arbitrarily, but the logical application of the working hypothesis stated in this chapter leads (see Fig. 10) to the conclusion that the direction of rotation of the filaments and the direction of the current are related to each

other as the direction of rotation and forward travel of an ordinary corkscrew.

Let us now examine a simple electric circuit, first in the nomenclature of the physicist, in terms of potential difference, amperes, and ohms, and second in the light of our analogy.

We shall assume that G is a direct-current generator impressing 100 volts on AB, a non-inductive resistance of 10 ohms. V is a voltmeter which requires only a negligible current to give a reading of 100 when placed across AB. If the leads GA and GB are good conductors and of large cross-section, the voltmeter will also read 100 when applied to the terminals of the generator.

The ammeter A' will read 10 amperes.

If AB is a uniform wire or rod, and C is its middle, the voltmeter in the position shown will register 50 volts.

The energy expended will be $EI = I^2R = 1,000$ watts, and in the example shown will be entirely absorbed in heating the resistance AB. With the polarity shown, the current will be flowing from the generator terminal to B, to A, and back to G.

As C is moved nearer A, the reading of the voltmeter will become smaller and smaller, reading 25 when three-quarters of the distance from B to A, reading 10 when nine-tenths of the distance, these readings being in accordance with the law of the "fall of potential" around a circuit.

Let us take the above statements and translate them into the language of our analogy.

The generator applies a torque to the ends of the circuit *GBA* in such a manner as to cause the molecular filaments of the circuit to rotate, this rotation being opposed by a resistance akin to friction in the portion *BA*, of such magnitude that heat is produced. The torque indicator (voltmeter) is so sensitive that no appreciable energy is required for its indication. (See Chapter IX.)

If the leads *GA* and *GB* are short and of large cross-section, the energy loss by resistance in them will be so small as to make no difference in the torque indication whether taken across *AB* or directly at the generator terminals.

The reading of the current meter is a number, and with the idea of angular velocity in mind we shall, for the present, accept that number as being simply a cardinal factor in the energy equation.

If the torque indicator reads 100 across *AB*, it will read 50 in the position shown in the sketch, for the angular stress on a molecular fiber will be one-half as great between the middle and either end as between the two ends, because the torque of resistance is exerted uniformly throughout the length of each fiber. This proportionality will hold for fractional points along the wire or rod.

The leads *GA* and *GB*, as stated above, are large in cross-section and good conductors, and hence show no energy loss, because the stress per molecular fiber is not appreciable.

The 1,000 watts, or 1.34 horsepower, which is being dissipated in heating the conductor *AB* is

accounted for by the logical assumption of work done in overcoming resistance to the rotation of molecular filaments.

Current direction means, in this case, that if a cross-section of the conductor be taken at *D*, the direction of rotation of the molecular filaments is clockwise when viewed in the direction of the arrow.

The reference to the "fall of potential" around a circuit leads us to inquire for an analogous phrase. Any conclusion drawn in this respect must be in harmony with the statement of our analogy and with our working hypothesis. Study of the former indicates that *in any homogeneous conductor of uniform size the stress on each molecular filament is met and counterbalanced by resistance to rotation, and this resistance is uniformly distributed throughout the length of the filament.*

The above statement may be applied to the parts of a non-homogeneous circuit, as, for example, one of copper wire and iron wire in series.

Greater resistance will be overcome in the poorer conductor, and the torque indicator (voltmeter) will show a greater reading when applied to the iron part of the circuit than when applied to the copper part.

ELECTRIC BATTERIES

An electric generator which derives its electromotive force from the cutting of lines of magnetic flux by conductors requires a prime mover, and so becomes an uneconomical machine in small sizes.

We are led, therefore, to inquire if there is not some other means of instituting molecular torque. Chemical action is immediately suggested, for chemical energy is liberated through the rearrangement of molecules. It is obvious that motion must accompany this rearrangement of molecules, and an assumption of rotation seems not illogical.

This prediction is justified by the general use of elementary chemical generators called batteries.

Commercial batteries are of two general classes, primary and secondary, the more common name of the latter being storage batteries. For portable batteries, the dry cell has been developed. The elements of which it is composed are carbon, zinc, and a solution of ammonium chloride made into a paste, with inert ingredients such as sawdust. The "dry cell" is merely a semi-wet battery. Wet batteries are used where portability is not a requisite, and their elements may be replaced when chemical action has destroyed any of them.

In most primary batteries zinc is the material which is actively attacked by the solution in which the plates are immersed. This solution is called the electrolyte, and in some batteries more than one substance is used in the electrolyte to counteract secondary chemical actions which hinder the main work of developing voltage.

Storage or secondary batteries are those in which chemical action is reversible. Current is passed through the plates and solution in one direction, causing certain changes in them which are automati-

cally reversed when the battery is used to supply current.

One type of storage battery has lead plates in an electrolyte of sulphuric acid; the Edison battery has plates of nickel hydroxide and iron oxides, and uses a solution of potassium hydroxide for electrolyte. The latter type of battery has considerable advantage over the lead type in that it may be neglected and abused by unskilled persons without damaging its capacity for absorbing and giving off energy. The lead type of cell has a greater immediate reserve capacity and consequently is preferred in some cases but it must be watched carefully on charge and discharge to prevent serious damage to the plates.

Storage cells are rated in ampere hours. The ampere-hour capacity of a lead cell is always computed, unless otherwise specified, on the basis of 8-hour discharge. An 80 ampere-hour cell will give 10 amperes for 8 hours, and 10 amperes is the normal charging current. Of course, a greater or smaller current can be taken from the battery, but the ampere-hour capacity will be different. For example, if a higher rate be chosen and 20 amperes taken, the battery will only last about 3 hours; whereas if 5 amperes only are taken, the ampere-hour capacity will apparently be increased, and it will stand up for perhaps 20 hours.

This makes the 8-hour rating a very important point to know in buying or using lead storage batteries. Edison batteries are designed and rated ordinarily on a 7-hour charge and 5-hour discharge.

There are listed below the voltages of some common cells.

Type of Cell	Voltage
Dry cell,	1.45
Leclanche wet cell,	1.45
Lead storage,	charged 2.5
	discharged 1.8
Edison storage,	charged 1.85
	discharged 1.00

The only reliable test of the state of charge of a lead-acid battery is the density reading of its electrolyte. On full charge in a healthy battery, the density should be 1.25 to 1.30, and on discharge should not fall below 1.15 to 1.18. Hydrometer readings must be taken carefully with respect to the temperature of the electrolyte. The densities here given correspond to a temperature of 60° Fahr. A gain of 0.001 is caused by each 3° *drop* in temperature, and conversely, a loss of 0.001 is found to follow an *increase* of 3° in temperature.

CHAPTER IV

Generation of Current in an Armature Coil—Current alternating as generated. Sine-Curve Representation of Alternating Current. Changing Alternating Current of Armature to Direct Current at Machine Terminals. Principle of Commutator. Theory of Direct-Current Motors.

STUDENT'S GUIDE

The manner in which the electric generator develops electromotive force is deduced in this chapter by means of the application of our working hypothesis. It is shown that the induced currents of the armature are alternating, and the analogy is employed to reveal the use and operation of the commutator in rectifying the alternating current so that direct current may be supplied. A brief exposition of the theory of the direct-current motor is appended.

HOW PROCESS SUGGESTED BY ANALOGY IS INSTITUTED AND MAINTAINED

It is well known that when a closed loop of copper wire is revolved in a magnetic field, a current of electricity is set up and maintained in the wire. When many of these coils are wound upon a spool which revolves in a magnetic field so that the coils of wire cut the lines of force in the proper manner, the combination is called an armature. By what process is a current generated in the armature coil? Our analogy

calls for the revolution of the lines of molecules in a
conductor transmitting energy.   It also suggests the
following possible way in which such revolution may
be originated and maintained in the conductor.

In Fig. 10, let *N* and *S* be the north and south poles
respectively of a field magnet.   The space *ABCD* is
a magnetic field of force, in which the lines of force
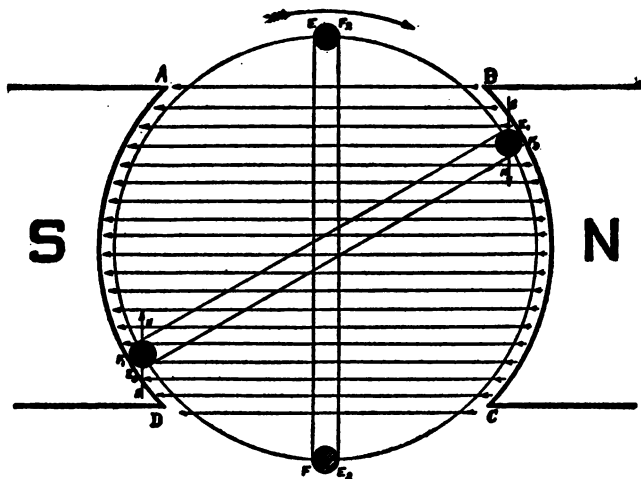
FIG. 10

are approximately parallel to *AB* and *DC*.   Let *E* and
F represent cross-sections of a closed coil of wire, re-
volvable about a central axis perpendicular to the
plane of section.   When such revolution takes place,
lines of force in the field are cut by the coil, and a
current is set up in the wire.   As we know that some
process goes on in the wire, we will assume as a work-
ing hypothesis that while the wire is cutting the lines

of force in the field *ABCD*, each molecule in the field is magnetized with its magnetic axis at right angles to the lines of force, and its north pole in the general direction of the motion of the coil at the point where the molecule is located.

In Fig. 10, let *E* now represent a particular molecule in one side or branch of the coil, and *F* another molecule in the opposite side of the coil.   *E* and *F,* with their subscripts, represent the same molecule, but in different positions of the coil. As *E* is carried by the revolution of the coil to E_1, we shall find *F* at F_1. By our hypothesis the molecule in the position E_1 is now being magnetized with its magnetic poles as shown by the arrow *SN*, while the molecule at F_1 will have its poles as also shown by its arrow *SN*. In each case the polar axis is at right angles to the lines of force of the magnetic field, and the north pole is in the general direction of the motion of the wire at the point where the molecule is situated. Each of these tiny magnets E_1 and F_1, being under the influence of the large field magnet *NS*, will tend to revolve rapidly and in such a direction as to bring the north pole of the molecule toward the south pole of the field magnet, and vice versa.

Looking at the figure with this in mind, the molecules E_1 and F_1, which are supposed to be both in and forming a part of the same flexible shaft, appear to be revolving in opposite directions; but as these two molecules are in opposite branches or sides of the same coil, they are really revolving correctly as a part of the same flexible revolving shaft. This can be illustrated

by bending a flexible shaft or even a small cord into a loop STU (Fig. 12) and twisting the cord between the thumb and finger at T. Looking from T toward E_1 and F_1 (Fig. 12), the cord will appear to revolve in opposite directions at the points E_1 and F_1, but will really be revolving throughout its length harmoniously in the same direction.

When the coil in Fig. 10 has revolved through 180° from its first position EF, so that the molecule E has come to the position E_2 and F to F_2, the coil ceases to cut lines of force of the magnetic field. As the revolution of the coil proceeds, it begins to cut again the lines of force; but when the molecule E has come to a position E_3 and F to F_3, these molecules will obviously be each revolving in a direction opposite to its former motion. The rate at which lines of force are cut by the revolving coil is greatest when the coil is 90° from the position EF. This rate, starting from zero at EF, varies as the sine of the angle through which the coil has revolved from EF. This appears at once from the inspection of the figure. Thus, following our analogy in connection with the above hypothesis, we are led to the conclusion that the electric current is alternating in the armature, and that its direction changes as often as the plane of the coil comes into the position EF or E_2F_2; that is, when the plane of the coil is perpendicular to the lines of force of the field.

The electromotive force thus generated may conveniently be indicated as in Fig. 11, where the ordinates of the curve represent the instantaneous values of the e.m.f. and the abscissæ the corresponding angu-

lar positions of the rotating coil. This is the so called sine-wave, the properties and construction of which are discussed in the Appendix.

It should be noted that in the solution of the alternating-current problems the sine-wave is taken to represent the instantaneous values of current and voltage because calculations are thereby made simple, and because commercial alternators do, in fact, give voltage values very closely approximating the theoreti-
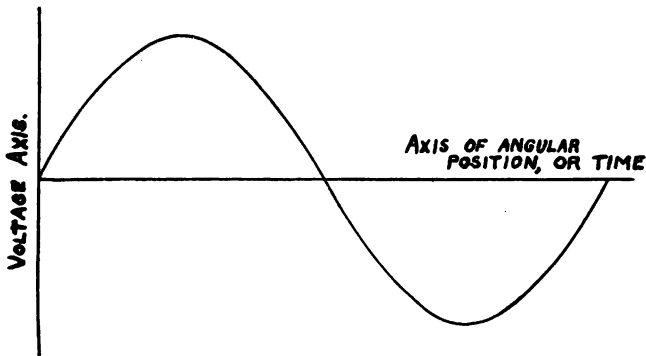
FIG. 11

cal. But the irregularities in the distribution of the coils about the armature, and variations of magnetic flux distribution in the air gap, sometimes cause variations from the true sine-wave form. And when a pure sine-wave voltage is impressed upon an inductance coil having an iron core, the current has a wave form considerably distorted from the sine-curve.

In the Appendix will be found definitions of *form factor* and *peak factor*. These quantities are measures of the variation of the wave from the sine form.

THE COMMUTATOR

Having shown that the current as generated in the dynamo is always alternating, we refer to our analogy to show how a direct current is obtained in the external
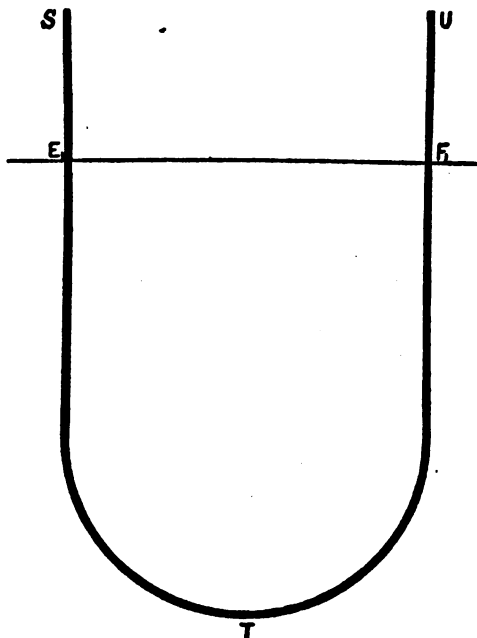
FIG. 12

circuit, from the alternating current generated in the armature of the dynamo.

In Fig. 13, let *ABCE* be an endless flexible shaft. If this shaft is given a rotary motion at any point as *A*, the shaft will rotate through its whole length. This illustrates a direct current of electricity. If the rotary

motion induced at A be first in one direction and then in the opposite, this alternating motion at A will obtain throughout the whole shaft, and we shall have illustrated an alternating current in $ABCE$. Now
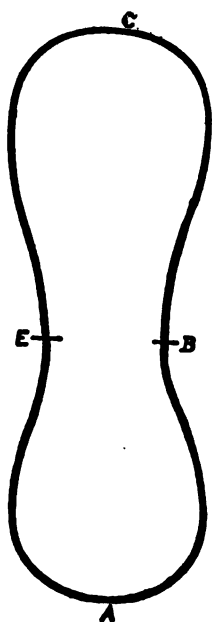
FIG. 13
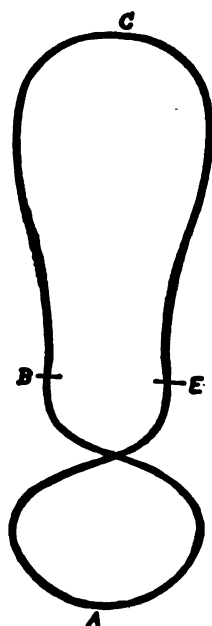
FIG. 14

suppose that at E and B there are detachable connections.

In Fig. 13, let the rotation at A be in a given direction. Then the rotation of BCE will be in the same direction as the rotation at A. Now let the connections at E and B be detached and E and B change places with reference to the ends of the part C, as

shown in Fig. 14; and at the same instant let the direc-
tion of the rotation at A be changed. Then it readily
appears that while the rotation at A alternates, the
rotation of BCE will continue in the same direction
as before. This interchange back and forth between
E and B, occurring as often as and simultaneously
with the change in direction of rotation at A, will give
a continuous rotary motion in the coil C; or in other
words, the alternating current at A has been changed
into a direct current in the external circuit C. This
reveals at once the principle of the commutator.

The continuous repetition of changes back and
forth from EB to BE is secured in the direct-current
dynamo by the proper connections of the armature
wires with the commutator sections. The direct
current is taken from the commutator by the brushes,
which are connected with the external circuit con-
ductors.

The principle of the commutator enables one to
understand the rotary converter, which is sub-
stantially an armature receiving by means of sliding
contacts alternating current from any source and
changing it into direct current, or vice versa, in the
same manner as the commutator of the dynamo
changes the alternating current received from the
armature wires into the direct current of the external
circuit.

THEORY OF DIRECT-CURRENT MOTORS

If an electric current is established in a conductor
lying in a magnetic field, mechanical force will be

Figure 16 is a section of the armature and a pole-
face, with the magnetic field represented by lines in
the air gap. A single conductor *P* is shown.   If the
direction of rotation of filaments is left handed, with
the magnetic polarity shown, application of the work-
ing hypothesis (left-hand rule, see Chapter V) gives
the direction of rotation of the armature shown by
the arrow.

Now conductor *P* is cutting magnetic lines and
therefore must have induced in it an electromotive
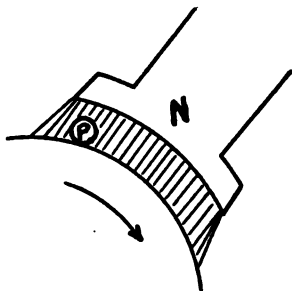
Fig. 16

force.   The working hypothesis (right-hand rule)
gives the direction of this induced torque as right-
hand rotation, and so we see that the induced voltage
is tending to oppose the voltage which is sending the
motor current through *P* in the left-hand direction.
Suppose that a load is suddenly removed from the
motor; there will then be an excess of mechanical
torque, and the armature speed will increase until the
counter e.m.f. has reduced the current to the value
just large enough to supply the new reduced me-

chanical torque. When the load is applied again, the
armature speed drops, the back e.m.f. is thereby
reduced, and more armature current flows to provide
the needed mechanical torque. In this way the back
e.m.f. is always opposing the applied voltage, acting
as an automatic resistance, adjusting itself to load
requirements.

Current taken by the armature is represented by
the following expression

$$I_a = \frac{E_L - E_a}{R_a}$$

where

I_a = armature current;
E_L = voltage at brushes;
E_a = counter e.m.f. of armature;
R_a = resistance of armature.

CHAPTER V

Induction—Relation of magnetism and electricity; relation of
induced to primary voltage; hand rules for determining
direction of induced effects. Self-Induction—Example;
response of molecular filaments to changes in strength
of magnetic field. Transformer—examples of use; voltage,
current, and energy relations. Induction Coil—Alternating
current induced by pulsating direct current.

Student's Guide

Self and mutual induction are the main headings
for this chapter. We here expand our knowledge of
the function of magnetism in electrical phenomena,
and learn that the molecular filaments of a conductor
are at all times subject to very delicate control by
magnetic lines of force. Illustrations chosen are the
telephone, the transformer, and the induction coil.

Induction

When a conductor in circuit and a magnetic field of
force are in such mutual relations that the conductor
moves relatively to the lines of force in such a manner
as to cut these lines, a current of electricity is gen-
erated in the conductor. This action is called induc-
tion. The lines of force may be constant and the
conductor move across them, or the conductor may
be at rest but in a fluctuating field of force.

In all cases the statement of our working hypothe-
sis holds—namely, when a conductor in a magnetic

field of force cuts these lines of force, the molecules
of the conductor constantly tend to form magnetic
poles, their instantaneous polar axes being at right
angles to the lines of force of the magnetic field, and
with the north pole in the general direction of the
motion of the conductor relatively to the magnetic
lines of force. By means of this hypothesis we may
readily explain the phenomena of induction.

Referring to Chapter I, we find that a conductor
carrying a current seems to be surrounded by lines
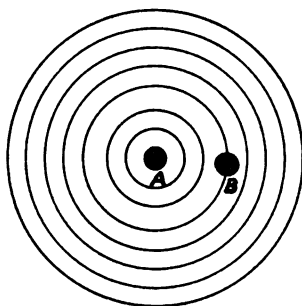
FIG. 17

of magnetic force; a magnetic needle placed near it
always tends to stand at right angles to the conductor.
If the current changes direction, the magnetic needle
instantly tends to change its direction through 180°.
If our assumption is correct that the molecules of the
conductor are constantly magnetized as stated, then
the effect upon these molecules in the presence of a
conductor carrying a current will be the same as the
effect upon a magnetic needle.

In Fig. 17 let *A* be a cross-section of a conductor
carrying an alternating current, and let *B* be a con-
ductor in another circuit, the two conductors *A* and
B being parallel and near each other.   Let the con-
centric circles about *A* represent the magnetic lines
of force which we know to exist around a conductor.
Since the current in *A* is alternating, these lines of
force will go from zero to their maximum distance
from *A* and back to zero, as often as the current alter-
nates.   In consequence of this, a current will be set
up in conductor *B*, usually called an induced current.
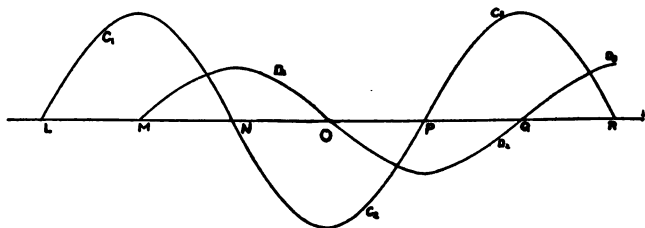
FIG. 18

What follows explains how the e.m.f. of this induced
current may be generated, and why it is 90° behind
the electromotive force of the conductor *A*.

Let Fig. 18 represent the electromotive forces of
the current in the conductor *A* and of the induced
current in the conductor *B*, in Fig. 17, and explained
above.   The letters *L, M, N, O, P, Q*, in Fig. 18 repre-
sent periods in the cycle 90° apart, with *L* beginning
at zero.   Let C_1, C_2, C_3, represent the electromotive
force of the current in *A*.   As the electromotive force
goes from the point C_1 at the time *M* (90° from the
beginning of the cycle) to zero degrees at time *N*

(180° from the beginning of the cycle), the lines of force from the conductor A of Fig. 17 cut across the conductor B of Fig. 17 as they would if B moved towards the circumference of these lines relatively to the lines; and consequently the molecules of the conductor B are magnetized with their polar instantaneous axes at right angles to the lines of force, and the north poles of these molecules pointing away from A. The molecules of B therefore tend constantly to rotate into a position with their polar axes at right angles to the conductor A. Consequently a current will be set up in the conductor B, whose electromotive force is represented by the line D_1, D_2, D_3, in Fig. 18.

At the point marked N in Fig. 18 (180° from the beginning of the cycle) the current in A alternates, and also the motion of the conductor B relatively to the lines of force around A, Fig. 17, changes direction. The alternation of the current changes the polarity of the conductor A, and the polar axes of the molecules of the conductor B also change through 180° at the same instant. And similarly at the point P in Fig. 18. Therefore the electromotive force in conductor B in Fig. 17 will be represented in Fig. 18 by the line D_1, D_2, D_3; or in other words, the molecules in the conductor B will continue to rotate in the same direction from M to O, or during the period from 90° to 270° of the cycle; and similarly, while the electromotive force of conductor A passes from O to Q, the molecules in conductor B will rotate in the opposite direction. Hence the electromotive force of the induced current is 90° behind that of the original current.

It will be well at this point to state two rules which have been found to apply to the phenomena of induction. It is highly important that we be able to predetermine the direction of induced currents. Ordinarily the student of electricity is cautioned to think only of induced electromotive force or voltage, current itself never being said to be induced, but always considered as resulting from the action of voltage on a resistance. But the definition of electricity as a means of transmitting energy, and the conception of the generation of current by the forcible rotation of material molecular filaments, make the distinction less important and consequently we shall use the phrases "induced voltage" and "induced current" as may best suit our convenience.

There are two "hand rules" which are helpful in applying our working hypothesis when determining the direction of induced current in generator action, and the direction of motion when motor action is taking place. The "right-hand rule" applies to the generator, or to any case of induced current where it is possible to determine the relative motion of conductor and lines of force.

Right-Hand Rule—Place the right hand in the magnetic field with thumb and fingers at right angles and extended, so that the lines of force enter the palm of the hand, with the thumb indicating the direction of relative motion of the conductor; the fingers will then indicate the direction of the induced current.

For motor action the "left-hand rule" applies.

Left-Hand Rule—Place the left hand in the mag-

netic field with thumb and fingers at right angles and extended, so that the lines of force enter the palm of the hand, with the fingers indicating the direction of the current; the thumb will then indicate the direction of the mechanical force exerted on the conductor.

SELF-INDUCTION

In the preceding discussion the assumption has been made that two circuits were always concerned, one having a current flowing in it producing a magnetic flux which, linking with the second circuit, induced therein a current. The phenomenon of induction is also observed in a single circuit whenever the current strength changes, for with every change of current a change of magnetic field strength occurs, the circuit is cut by its own lines of magnetic force, and is subject to the several laws of induction.

The practical effect of this action may be observed on breaking the current in a highly self-inductive circuit—that is, a circuit in which the mechanical arrangement is such as to allow many magnetic lines to cut the conductors of the circuit. The change which is being effected is a decrease of current, and the torque with which self-induction opposes this change causes the arcing at the switch points which is characteristic of inductive circuits, such as the field windings of generators. If we were dealing with the rotation of flexible shafting of finite size, the tendency to continue in motion would without hesitation be ascribed to inertia; and we are thus inclined to endow

our molecular filaments with a property of the nature
of inertia.   But we must recognize the fact that it is
not a phenomenon comparable with that of a body of
sensible dimensions, for it is a fact of common experi-
ence that the voltage which appears at the terminals
of a field circuit when the field current is suddenly
interrupted is often many times greater than the
voltage impressed on the field when the normal field
current is flowing without interruption.

The energy thus exhibited is stored in the magnetic
field while the current is flowing steadily in the coils
and is not due to any tendency of the filaments to
continue their motion, but to a true electromotive
force or torque which is generated in the manner
described above, because of the collapsing of the lines
of force upon the turns of the field coils, after the
switch is opened.

It is fitting at this time to point out the fluidity of
motion which the filaments of a good conductor
possess.   They are instantly and definitely responsive
to any torque impressed by the slightest variation of
the magnetic field.   This relation is shown very clearly
in the operation of the telephone transmitter, which
consists of a soft iron diaphragm mounted near the
poles of a horseshoe-type permanent magnet which
is provided with coils of wire wound on the poles.   The
attraction of the magnet holds the diaphragm in
slight tension.   When a sound disturbs the diaphragm
and causes it to vibrate, the magnetic field strength
is varied as the diaphragm approaches or recedes
from the magnet poles.   This variation of field causes

a cutting of the turns of wire in the coils by lines of force, and currents are caused to flow in the coils and in the telephone line. The receiver at the other end of the line is constructed similarly, and the reverse process is instituted, causing the diaphragm to vibrate in unison with that of the transmitter and to reproduce the sound which caused the original vibrations. The transmitter as used to-day is not of the type described above, but the receiver has not been modified except in form and arrangement of the parts described.

The foregoing discussion suggests the utilization of the principle of induction in an apparatus which, by an increase or decrease of the number of conductors in the circuit in which the induced current flows, can raise or lower the voltage as desired, with a consequent change of the angular velocity (or current) to keep the energy equation balanced. The effect of the transformer is like that of the spur and pinion of mechanical transmission systems.

If in Fig. 17 we place another conductor C beside B, as in Fig. 19, there will be induced in it an electromotive force exactly equal and in phase with that induced in B. Should these two conductors be connected in series, we should have a two to one, step-up transformer action. The current in BC will be one-half that in A, in order that the energy relation may be a true one.

The distribution of energy in lighting a city is accomplished by means of transformers. In a specific case the hydroelectric generating plant is 200 miles

from the cities where the energy is used. The genera-
tors operate at 2,300 volts, and their output is com-
municated to the transmission line by means of
transformers which change the voltage to 100,000,
with, of course, a proportionate drop in current.

It is the lowering of current which is desired, for
the line losses due to heating are proportional to the
square of the current.

On the outskirts of the cities there are transforming
stations where the reverse is accomplished, the trans-
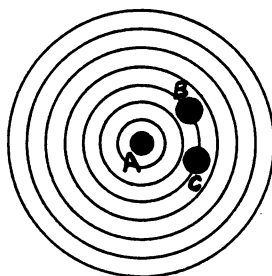
FIG. 19

formation being from 100,000 down to 2,200 volts,
for it would be dangerous to carry the higher voltage
through the streets and into buildings. Each con-
sumer of energy then has his own smaller transformer,
which further reduces the voltage to 220 for motors
and 110 for house lighting.

The facts to be remembered about transformers
are, that the ratio of transformation of voltage is
directly as the number of turns in primary and
secondary coils; that if voltage is raised, current is

lowered; and that some energy is always lost through heating the conductors and the iron core on which they are wound.

These relations are expressed thus:

N_1 = number of turns in primary winding;
N_2 = number of turns in secondary winding;
I_1 = current in primary;
I_2 = current in secondary;
E_1 = voltage impressed on primary;
E_2 = voltage induced in secondary;
W_1 = energy input;
W_e = energy lost in heating transformer;
W_2 = energy taken from secondary.

$$\frac{N_1}{N_2} = \frac{E_1}{E_2} \qquad E_1 I_1 = E_2 I_2$$

(assuming non-inductive load and neglecting losses).

$$W_1 = W_e + W_2$$

$$\text{Efficiency} = \frac{W_2}{W_1}$$

Voltage and current relations in a transformer are related by the laws stated at the beginning of this chapter. (See Chapter VI for transformer connections.)

The induction coil is a common and useful piece of apparatus in which a pulsating direct current of low voltage is converted to an alternating current of high voltage. Figure 20 shows the circuits of a simple induction coil.

C is a soft iron core on which are wound perhaps one hundred turns of medium-size wire P, called the

primary winding. One end of the primary is brought out through I, a vibrating armature type of interrupter which causes rapid interruption of the primary current. S is the secondary winding which consists of many hundreds of turns of fine wire wound over the primary turns.

When the current is first established in the primary, lines of force spread from the turns of the primary coil and, in doing so, cut the turns of the secondary, thus inducing a high voltage which appears at the terminals of the secondary winding. The armature
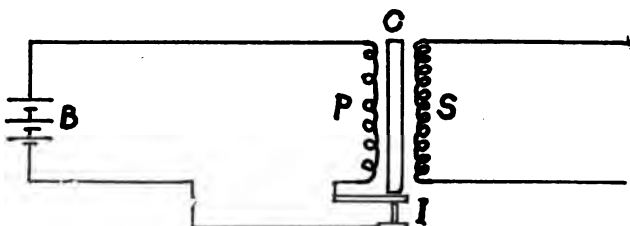
Fig. 20

is attracted by the core C, thus interrupting the current and causing the collapse of the lines of force. In collapsing they cut the secondary turns in the opposite direction, and the electromotive force at the terminals is equal to that induced when the current was established, but opposite in direction. Consequently for each to and fro motion of the armature or interrupter, a complete cycle of alternating electromotive force is induced in the secondary winding. The relation between current in the primary $\dot{C}$ and the induced secondary voltage E is shown in Fig. 21.

In order that the student may have some familiarity

with the dimensions and materials necessary for con-
structing the above described coil, there is given here
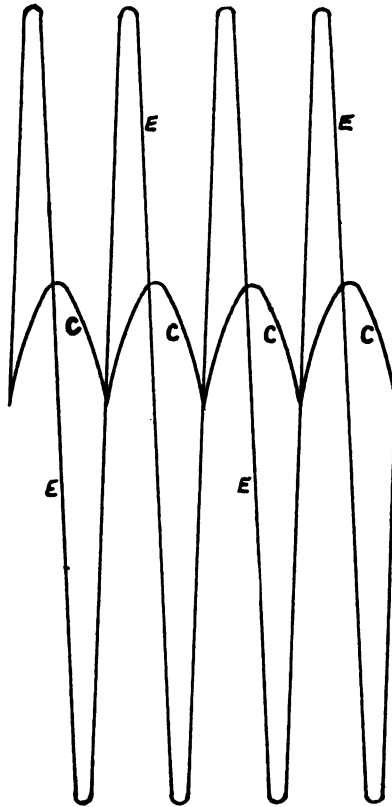
FIG. 21

a list of the parts of a coil which will produce at its
secondary terminals a spark 3 inches long. Core:
bundle of iron wire, annealed No. 20, 1¼ inches in

diameter, 13 inches long; primary, four layers of No. 12 double silk covered copper wire, about $4\frac{1}{2}$ pounds; secondary, 4 pounds of No. 36 double silk covered copper wire, about 28,000 feet.   This coil will have an effective voltage of approximately 100,000 volts at the secondary terminals.

CHAPTER VI

Combining Out of Phase Torques—Graphical solution; triangle relation between Z, R, and S. Power Factor—Adjustment of power factor by introduction of new torque; efficiency of this procedure. Three-Phase Circuits Developed—Three-phase alternator; transformers connected to three-phase alternator. Measurement of three-phase power—Two-wattmeter method.

STUDENT'S GUIDE

We have seen that several torques out of phase with each other may be acting on a circuit at once; in this chapter we shall show how to combine them. The relation between certain of these original and resultant torques is important enough to be named—we call this relation the power factor; and since it is desired to have the highest obtainable power factor, we outline means of establishing the desired relation. From the general case of out of phase torques it is but a step to the orderly arrangement of three-phase systems, and the chapter closes with a discussion of the means of measuring the power in three-phase transmission circuits.

COMBINING OUT OF PHASE TORQUES

An electric circuit may have both resistance and self-induction. Let us examine, from the viewpoint of our analogy, the characteristics of a circuit having both resistance and self-induction. In Fig. 22, let

the sine-curve L represent the e.m.f. or torque of the resistance or ohmic load. Let I represent the e.m.f. of self-induction, and let the current be a unit current. This unit current should have a maximum ordinate which gives the mean value equal to unity. The curves marked C represent unit currents. It has been
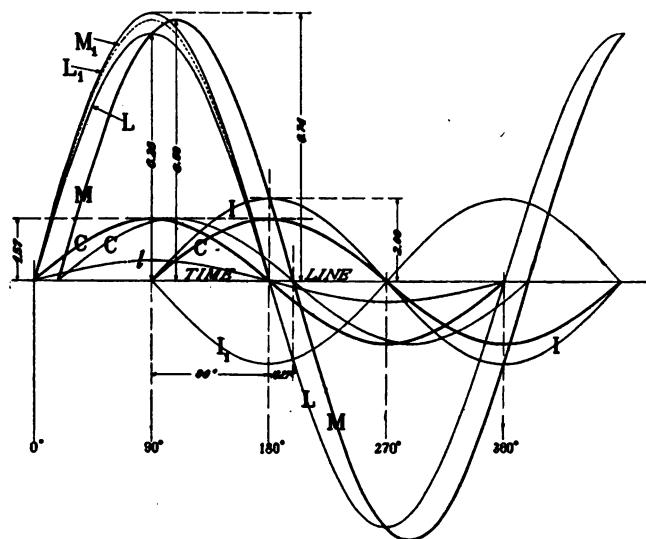

FIG. 22

shown that an induced e.m.f. is 90° behind the primary e.m.f. Therefore the curve I starts at 90° while the curve L starts at (zero degrees) 0°. Consider a line of shafting of negligible weight and acted upon by two torques that are out of phase with each other. If there is alternating reciprocating rotary motion of the shaft under the action of an impressed torque,

it follows that the impressed torque must be at every instant equal and opposite to the resultant of the two resisting torques, because action and reaction must be equal and opposite. It also follows that the angular velocity of the shaft in question must be in phase with the resultant torque.

Combine graphically the torques represented by L and I. The resulting curve M cuts the time line about $18.27°$ behind the point at which L cuts it. In other words, the resultant torque has been set back this number of degrees by the introduction of the out of
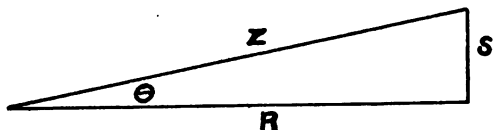
Fig. 23

phase torque of self-induction. The sine-curves L, M, and I are really energy curves, since the current is unity. Their areas represent respectively the impressed energy, the energy required to overcome the load, and the energy required to overcome self-induction, for half a cycle, and are proportional to the squares of the maximum ordinates of the curves. (See Appendix.) Denoting the maximum ordinates of L, I, and M by R, S, and Z respectively, we have $Z_2 = R_2 + S_2$, or $Z = \sqrt{R_2 + S_2}$. This equation is represented graphically by the triangle, Fig. 23.

In this triangle the angle between Z and R is usually termed the "lag angle." Let it be denoted by

θ. If we determine this angle by putting the values

of R and S from Fig. 22 into the equation $\tan \theta = \dfrac{S}{R}$,

we find that the value thus computed agrees substantially with the 18.27° shown in Fig. 22.

The meaning of the term "lag" evidently is that the current lags this number of degrees behind the e.m.f. of one of the component torques—namely, L—and not that it lags behind the resultant e.m.f., M.

The cosine of the so called lag angle is used as the power factor, and its use for this purpose is very convenient. The power factor is the relation of the useful work to the total energy expended. Calling R or the ohmic load the useful work, which is represented by the area of L, and Z the energy expended as represented by the area of M, we should have power

factor $= \dfrac{R}{Z}$, which gives the result obtained for θ

from the triangle, Fig. 23.

That any change in the load will affect θ is evident from the accompanying figures.

Figure 24 shows the addition of non-inductive load, with consequent decrease of θ. Figure 25 shows the diagram for a circuit having capacity and non-inductive resistance. Figure 26 shows a case of inductive effect overbalanced by capacity.

An important situation, which is discussed in the following pages, is illustrated by Fig. 27. Here, by the expenditure of a little additional energy, we are able to introduce into the circuit a torque exactly

equal and opposite to the torque of self-induction, thus bringing θ to zero, the power factor to unity, and increasing the output of the system.
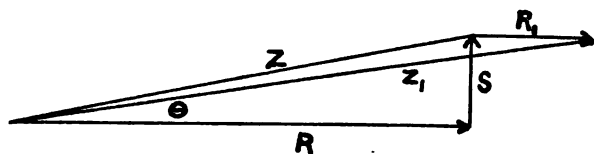
FIG. 24
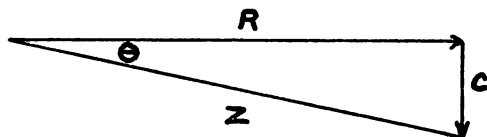
FIG. 25
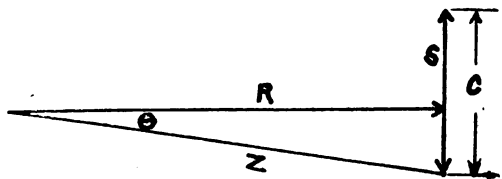
FIG. 26
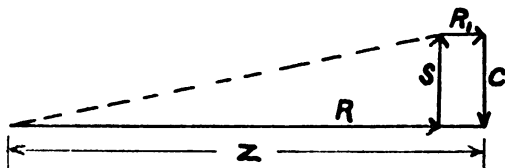
FIG. 27

The angle θ in the above figures — commonly termed the "lag" angle—is due to the self-induction

being out of phase with the load.   It is an indication
rather than a cause of the loss due to self-induction.

A way to bring the power factor to unity at once
suggests itself from the foregoing analysis.   Let Fig.
28 represent an alternating-current motor the arma-
ture coils of which form a part of the circuit.   Let the
field magnets of the motor be energized by a direct
current generated by a dynamo receiving power either
from an outside source or from the line itself.   If
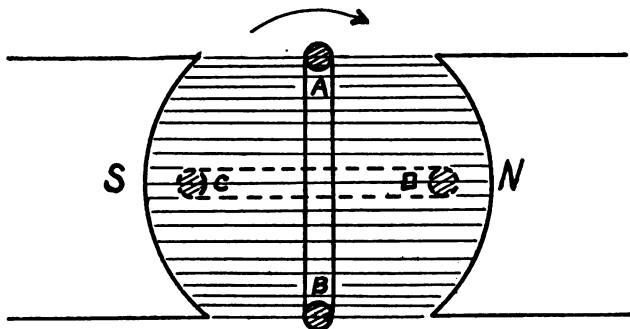
FIG. 28

when the coil is in the position A—B, its current and
e.m.f. are at a maximum, with the shafts at A having
counter-clockwise rotation, then when the coil is in
the position C—D, or 90° from A—B, the e.m.f. in-
duced by passing the coil through the magnetic field
between N and S will be at its maximum, the shafts
at D having clockwise rotation; and we shall have
introduced into the line a new torque which may be at
every instant equal in value and opposite in direction

to I, Fig. 22. The adjustment required to thus bring the line into synchronism consists in regulating the strength of the poles N, S by varying the exciting current. This is a crude illustration of a means for securing synchronism, but it makes clear the principle of the synchronous condenser on the line.

With such an arrangement as the above, the line is sometimes said to have capacity. The effect is such as would result from capacity, but the process is not in accord with our idea of capacity. (See Chapter VII.) The result of the condenser on the line as above described is shown in Fig. 22 by the curve I_1, which is equal to the curve I, but 180° behind I. Constructing now the curve representing the resultant e.m.f., we find it to be the curve L. The two e.m.f.'s 180° apart have balanced each other and eliminated the lag. This, however, has been done by an expenditure of energy. If energy is taken from the line to run a motor generator set which supplies direct current to energize the magnets N and S in Fig. 28, this will add to the load L. This additional load may be represented by a sine-curve l, Fig. 22, the area of which should be somewhat greater than the area M—L. To illustrate the effect of this additional load, assume that it is once and a half M—L, or $l = \frac{3}{2}(M$—L.) If now we proceed to construct the resultant e.m.f., we have the curve L_1. The line is now in synchronism and carrying a load and a resultant expenditure of energy greater than the original load by an amount equal to L_1—L and greater than M by

the amount L_1—M. The value of $\dfrac{(L_1-M)}{L_1-L}$ repre-

sents the efficiency of the motor generator set used to energize the field magnets of the condenser on the line.

Thus by expending an amount of energy equal to L_1—L we have produced isochronism, thus overcoming the effect of self-induction and raising the energy in the line from L to L_1 and increasing the net output from L to $M_1 = M$.

THE FOREGOING RESULTS ARE LOGICAL DEDUCTIONS FROM OUR ANALOGY AND WORKING HYPOTHESIS. THESE RESULTS ARE NOT, WE BELIEVE, ESSENTIALLY CONTRARY TO OR INCONSISTENT WITH THE LATEST KNOWLEDGE OR THE BEST PRACTICE OF ELECTRICAL ENGINEERING. THIS CHAPTER IS BUT ONE ILLUSTRATION OF THE PURELY DEDUCTIVE METHOD WE ARE ABLE TO EMPLOY. IT IS WORTHY OF NOTE THAT IN OUR DEDUCTIONS WE WERE NOT DEPENDENT UPON A PREVIOUS KNOWLEDGE OF THE SUBJECTS TREATED.

THREE-PHASE CIRCUITS

The copper required to install the complete circuit between generator or transformer and its load is a considerable item in the cost of electrical power transmission projects.

Let us take three transformers with a load for each and attempt to arrange them with respect to the least
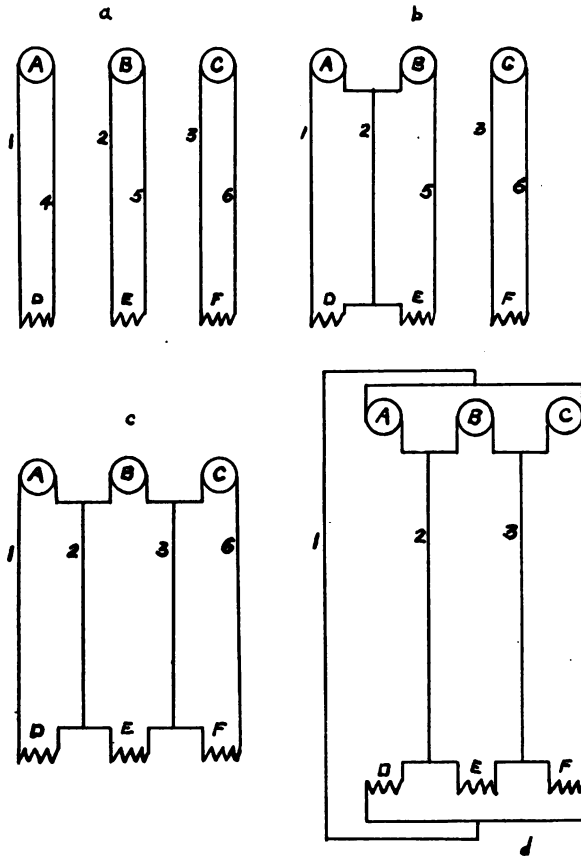
Fig. 29

possible outlay of copper. The elementary installation with three separate circuits is shown in Fig. 29,

where A, B, C represent the transformers, D, E, F their respective loads, and 1, 2, 3, 4, 5, 6 the copper wires composing the circuits.

In Fig. 29b, with proper regard for the current directions, wire number 4 has been omitted.
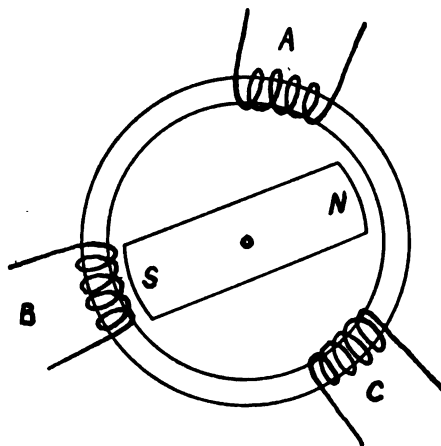
FIG. 30

In Fig. 29, a similar consolidation of circuits B—E and C—F has been effected.

In Fig. 29c, it will be seen that wires 1 and 6 are the only ones having but one connection to load and transformer, and Fig. 29d shows a consolidation involving the torque of all three transformers, making the circuit of three wires the equivalent of six.

The torques of the three transformers are adjusted to follow one another at 120° intervals by the distribution of conductors in the generator, as shown by the sketch, Fig. 30.

The generator coils, or phase windings as they are called, are indicated by A, B, and C. Their electromotive forces are represented by A, B, C of Fig. 31. The field marked N—S in Fig. 30 revolves inside the
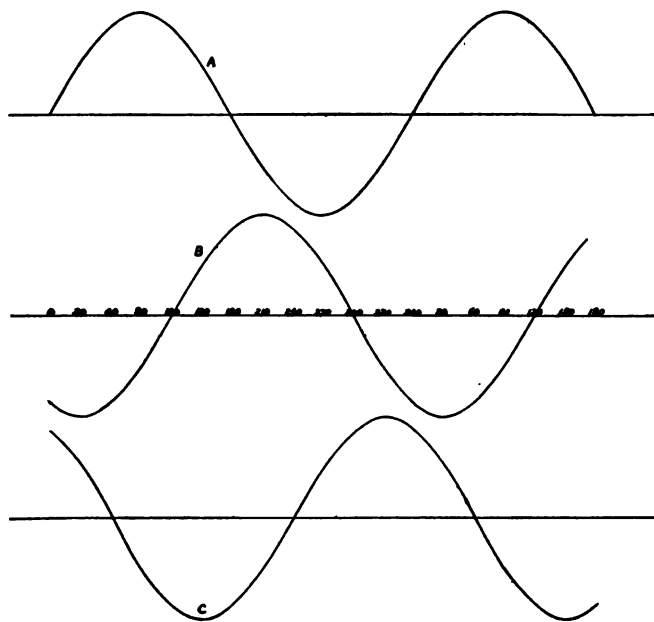
FIG. 31

stationary armature, thus allowing the load currents to be taken from A, B, C without moving contacts. The electromotive forces generated in the position shown in the sketch of the alternator circuits may be found from Fig. 31 at the 30° and 210° points; A and C are at one-half maximum in one direction, while B is at its maximum in the opposite direction.

It will be noted that at every instant the sum of the three voltages, taking their direction into account, is zero. Consequently there will not be any danger in short-circuiting the three transformers, as shown in Fig. 30. This connection is known as the delta.

The student will find it convenient to reproduce Fig. 31, and draw vertical lines through the degree points in the figure. These lines, wherever drawn, will show the amount and direction of the voltage in each of the alternator circuits.

Connection of alternator and transformers is shown in Fig. 32.

Three-phase connections are also made in the form known as the Y or star connection. The evolution of this type of connection is shown in Fig. 33.

Simple internal circuits of a transformer are shown in Fig. 34.

As was previously stated, the power in a single-phase alternating-current circuit is given by the expression $EI \cos \theta$, where

E = effective volts;

I = effective amperes;

θ = lag angle;

$\cos \theta$ = power factor.

From inspection of Fig. 32, it will be seen that the line voltage per phase in the delta system is the same as the voltage per phase in the alternator. The current per line, however, is not equal to the current per alternator phase, but will be found from Fig. 31 to be, making proper allowance for sign,

$$I_L = \sqrt{3}\, I,$$
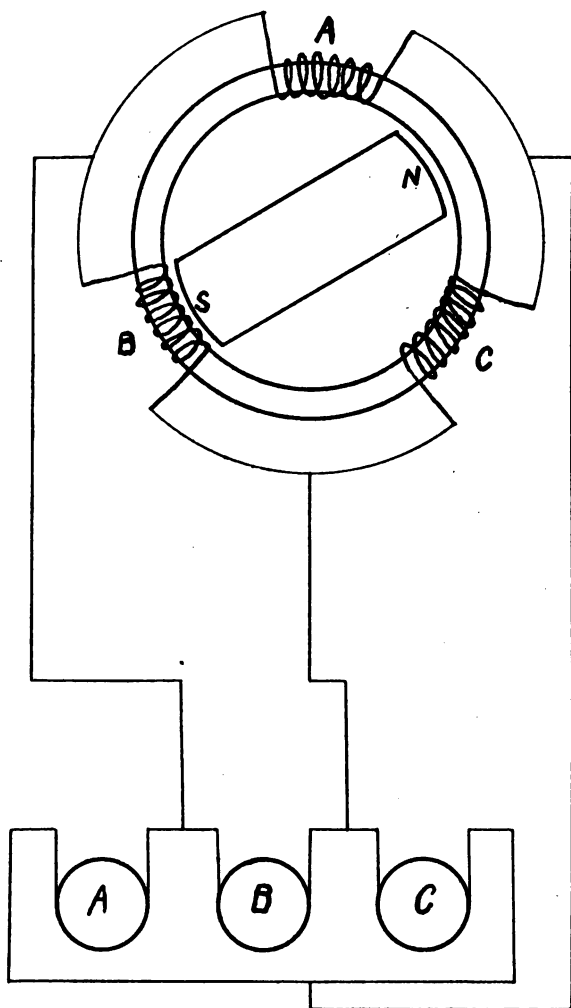
Fig. 32

Where

I_L = line current,

and

I_ϕ = current in the alternator phase,

effective values being used.
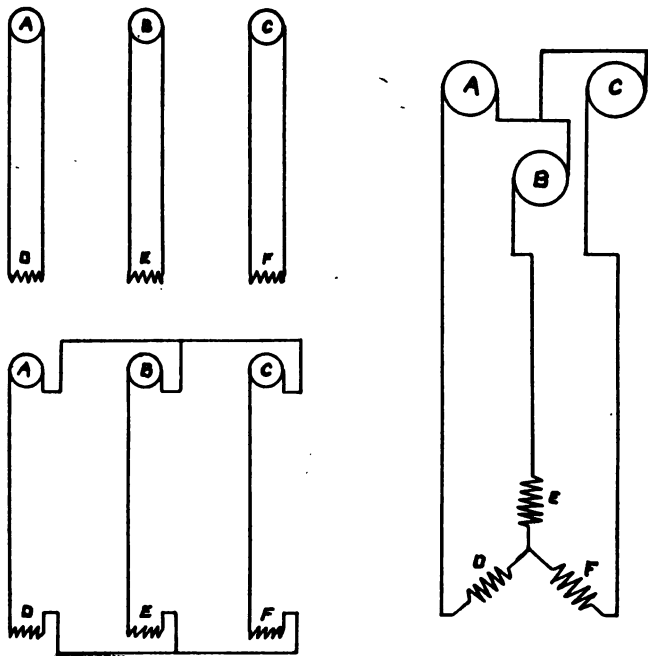
FIG. 33

Similarly in the Y or star system of connection, the current is seen to be the same in line and alternator phase, but the voltage of the line is

$$E_L = \sqrt{3}\, E_\phi$$

where

E_L = line voltage,
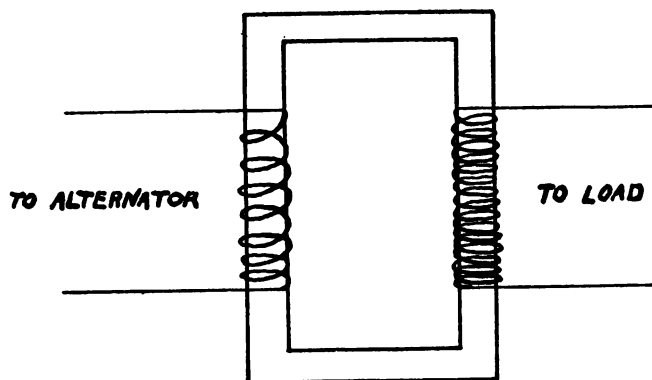
CORE TYPE
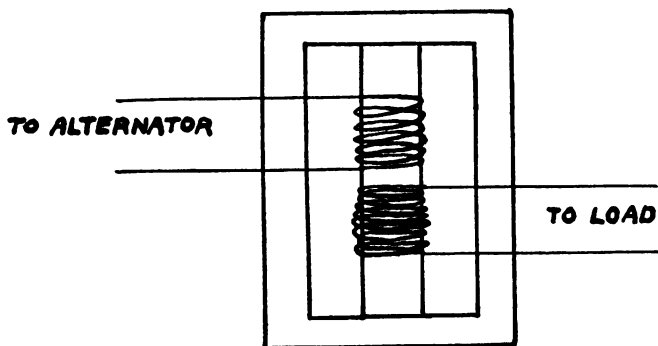
SHELL TYPE

FIG. 34

and

E_ϕ = phase voltage,

effective values being used.

Now in either system if the load is evenly distributed among the three phases, the power will be equal to

$$P = 3\,E_\phi\,I_\phi\cos\emptyset$$

where P = power, and other symbols have meanings given above. But in practice the line values and not the phase values are ordinarily known.

Hence from $E_L = \sqrt{3}\,E_\phi$, above,

$$E_\phi = \frac{E_L}{\sqrt{3}}$$

and

$$P = 3\,\frac{E_L}{\sqrt{3}}\,I_\phi\cos\emptyset$$

In the star system

$$I_\phi = I_L$$

and

$$P = \sqrt{3}\,E_L I_L\cos\emptyset$$

For the delta system

$$I_L = \sqrt{3}\,I_\phi$$
$$E_L = E_\phi$$

and

$$P = 3\,E_\phi\frac{I_L}{\sqrt{3}}\cos\emptyset = 3\,E_L\frac{I_L}{\sqrt{3}}\cos\emptyset$$

or, in both systems,

$$P = \sqrt{3}\,E_L I_L\cos\emptyset$$

Power is measured in practice by the use of watt-meters. Single-phase power may be measured by inserting the current coil of a wattmeter in one line, while the potential coil is connected across the line.

Three-phase power may be measured with two watt-meters connected as shown in Fig. 35.
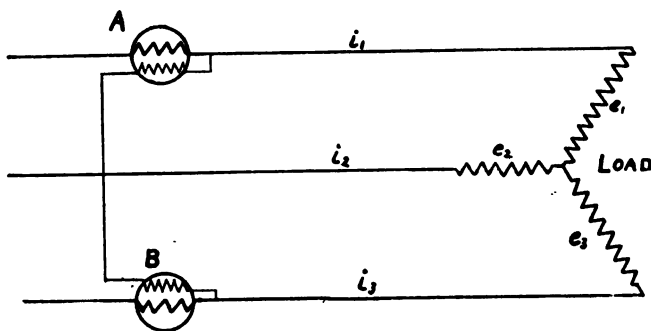
FIG. 35

The sum of the readings of the meters will give the total power when the lag angle is less than 30°, and their difference when the angle is greater than 30°.

The proof of the above follows:

P = total power at any moment;

e_1, e_2, e_3, and i_1, i_2, i_3, are the phase voltages and currents.

$P = e_1 i_1 + e_2 i_2 + e_3 i_3$ (instantaneous);

$i_1 + i_2 + i_3 = 0$ (see Fig. 31);

whence

$i_2 = -(i_1 + i_3);$

and substituting above,

$$P = e_1 i_1 e_2 (i_1 + i_3) + e_3 i_3;$$

$$P = (e_1 - e_2) i_1 + i_3 (e_3 - e_2).$$

Now $e_1 - e_2$ can be shown to be the voltage across the potential coil of meter A, and $e_3 - e_2$ is the voltage across the potential coil of meter B.

Also $(e_1 - e_2) \, i_1$ is proportional to the torque exerted on the moving element of meter A, and $(e_3 - e_2) \, i_3$ is proportional to the torque exerted on the moving element of meter $B;$ and hence by proper calibration of scale the sum of A and B will represent the total average power of the three phases, for the inertia of the meter pointers will automatically average the instantaneous values and will therefore read average power.

CHAPTER VII

The Condenser—Functions; analysis of condenser action; the
dielectric; current and energy relations; practical example.

STUDENT'S GUIDE

From the study of induction and its effects we pass
to the consideration of the condenser, an apparatus
which has operating characteristics exactly opposite
to those of induction, but which, when existing by
themselves in excess, are undesirable. The neutraliz-
ing effect of capacity and inductance is explained, and
a practical circuit containing both is discussed.

THE CONDENSER

A condenser is usually described as consisting of two
insulated conductors charged, one with positive, and
the other with negative electricity. If the two charged
conductors are connected, either by contact with each
other or through a third conductor, a current passes
which restores the electric equilibrium. The charged
conductors are said to be capable of containing a
certain quantity of electricity. The condenser is
said to have capacity. In the application of our
analogy, we have no occasion to recognize electricity
as an entity, and therefore we must inquire how
the analogy explains the phenomena of the electric
condenser.

Our analogy regards a conductor as containing an
indefinite number of infinitesimal shafts, "reasonably

rigid and elastic with regard to torsion." Under a
torsional stress, these infinitesimal shafts will have a
torsional strain, and being elastic, they will return to
their normal condition unless prevented by the
contact of a non-conductor. We therefore define a
charged condenser as two insulated conductors whose
lines of elementary flexible shafts are subject to
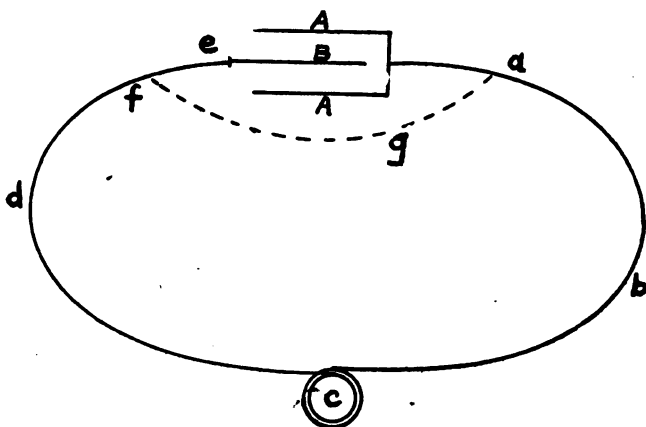
FIG. 36

torque. The material used to insulate the condenser
plates or conductors is termed a dielectric.

In Fig. 36, let *abcde* be a simple circuit with the
condenser composed of two copper plates *A*, *A*, and
the single plate *B*, the connections (except *a*, *g*, *f*)
being as shown. If the condenser were removed and
the circuit completed by joining the wires *a* and *f*,
the maximum voltage in the circuit with a given
current would be that due to the resistance of the

circuit. When the condenser is in the line (as in the figure, omitting *a*, *g*, *f*) the insulation between the plates *A*, *A*, and the plate *B* prevents a current passing between the plates. If an alternating electromotive force be applied at *c*, it will be transmitted to the plates *A*, *A*, and plate *B*. If the transmitting elementary shafts in the conductor were absolutely rigid, there could be no current in any part of the line; but as the elementary shafts are subject to torsional strain, and as they are elastic, all the elementary shafts in the plates will be subjected to a torsional alternating strain. This strain will be the greater because the insulation of the plates will permit a greater voltage in the circuit than was possible when the condenser was not in the line.

The torsional strain in the elastic shafts represents a condition of energy, which energy will be released when the strain is relieved. When the alternating voltage has been applied for a time at *c*, if the wires are disconnected at *a* and *f*, the condenser will be "charged." In this state it is said to contain a certain quantity of electricity. If connection is made between the plates, say by a conductor *a*, *g*, *f*, there will be a momentary current in this conductor. We attribute this to the release of the stress on the elementary shafts, thus allowing them by their elasticity to give up their stored energy due to the torsional strain to which they were subjected so long as they were insulated.

The operation of the condenser under the application of an alternating e.m.f. is to bring the torque in

the elementary shafts gradually to a maximum when the applied e.m.f. is a maximum. This is 90° from the zero of the e.m.f. At this point the e.m.f. begins to diminish and the energy stored in the shafts due to their torsional strain is gradually given off and the strain becomes zero again when the e.m.f. becomes zero. This process is repeated during the next 180°, but by the rotation of the shafts in the opposite direction. In long lines the resistance lowers the voltage. A condenser on the end of the line permits the voltage to be increased even to distant parts of the line. In the many elementary shafts in the plates of the condenser, energy is alternately stored and given off, and with increased voltage, as above described.

In our analogy the elementary flexible shafts in the dielectric are capable of receiving and transmitting an electromotive force (or torque) without evidence of current in the dielectric. In a dielectric, as well as in a conductor, the properties of the elementary flexible shafts must vary in different substances, particularly in reference to their rigidity, their elasticity, and their resistance.

If the elementary shafts are quite rigid in regard to torsion, they will have a comparatively small torsional strain under a given e.m.f. If at the same time these elementary shafts have low elasticity, they will give back but a small part of the energy required to produce in them a given torsional strain. If the elementary shafts have great resistance due to their mutual interference, a slight rotary motion or displacement will cause the resistance to balance the

applied e.m.f. This rotary motion may be so small as not to be ordinarily measurable as current.

Following our analogy, we must consider the lines of molecules in the dielectric (considering air for the present) as subject to torque. Air is not a complete non-conductor. When the e.m.f. is sufficient, current passes, always accompanied by an appearance of heat. It may be reasonably assumed that a very limited rotary motion of these elementary shafts takes place before the current is noticeable. If the condenser plates are very widely separated, the condenser effect is neutralized or greatly diminished. This is explained by considering that the resistance to the slight rotation of the lines of molecules in the dielectric (which extend far out into the atmosphere); the torsional strain, and the low elasticity of these elementary shafts, all combine to absorb most of the energy supplied to the condenser. Therefore, when the plates are separated, the energy is not stored in the elementary shafts of the condenser; in other words, the condenser cannot become "charged."

When the condenser plates are brought closer together, each short molecular shaft in the dielectric has a torque applied to each of its ends, and in the same direction; and while this short shaft rotates until the resistance of the dielectric balances the torque applied to it, there is but little energy absorbed in the dielectric, because the shafts between the plates of the condenser are so short.

If different substances are used for the dielectric, the efficiency of the condenser will vary with the

character of these different substances. Dielectrics
for condensers are classified with reference to their
several specific inductive capacities. The greater the
specific inductive capacity of the dielectric, the greater
the efficiency of the condenser.

Air has a less specific inductive capacity than glass;
that is, air is a better conductor of electricity than
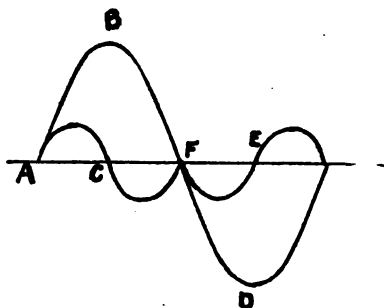
FIG. 37

glass, and therefore absorbs more of the applied energy
than glass when used as the dielectric of a condenser.

It is a matter of speculation how far the stress in the
elementary shafts in the air may extend, and at what
distances these lines may have slight rotary motion.
It does not seem an unreasonable stretch of imagina-
tion to suggest that there may be enough motion at
great distances to transmit wireless messages.

The graphical representation of e.m.f. current, and
energy in connection with the electric condenser,
follows.

We shall take a sine-wave impressed voltage and
attempt to find the logical current result.

If our condenser is uncharged, and if the electro-motive force be applied just as it is growing from zero to a positive maximum, at A, Fig. 37, a stress is placed upon the elastic molecular filaments of the condenser. This stress is resisted by the dielectric. Consequently a strain results in the molecular filaments of the con-
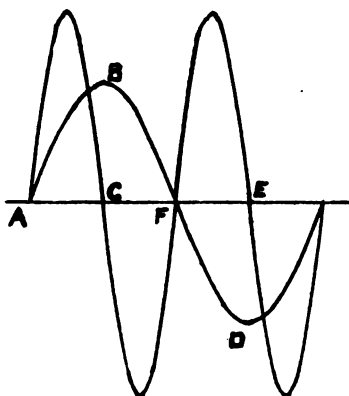
FIG. 38

denser which appears in the leads of the condenser as current. This current will increase until the torque reaches its maximum B, and will then commence to diminish, when the condenser filaments, being relieved of their maximum stress, will begin to rotate in the opposite direction at C, where there is a reversal of current direction, as shown. The current persists while the stress remains, but becomes zero at F when the torque dies to zero. From that point the action repeats, with FDE corresponding to AB and C.

The curve of energy is shown in Fig. 38.   Now if we
construct a similar diagram for a circuit containing
pure inductance, we shall find a clue to the means by
which capacity offsets the effect of inductance.

In Fig. 39, the current C is 90° behind the position
it would occupy if the circuit were non-inductive.
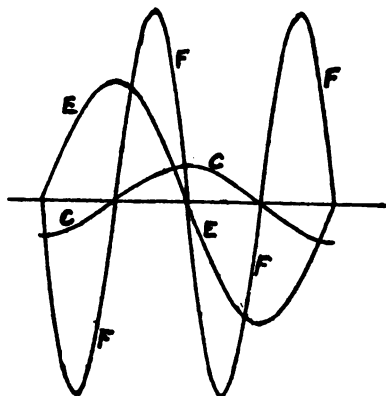
FIG. 39

The product of C by E is energy, and produces curve
F, which will be seen to be 180° displaced from the
energy curve of capacity, Fig. 38.   Hence we may
readily understand the opposite effects of capacity
and inductance.

The following example will serve to give a better
conception of the practical side of the matter.

If we impress an alternating voltage E on a circuit
consisting of a non-inductive resistance R, a certain
current C will result, and the rate of energy transmis-
sion we may call P.   This is represented by Fig. 40.

Let us now introduce an inductance S into the circuit, as shown in Fig. 41. C will be decreased, and P will be decreased. The inductance has introduced a torque of self-induction which has opposed the transmission of energy.
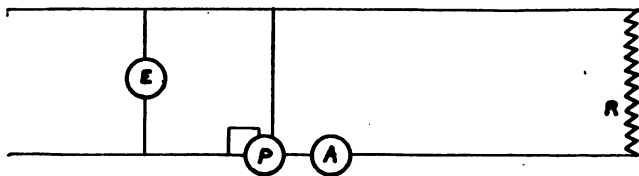
FIG. 40
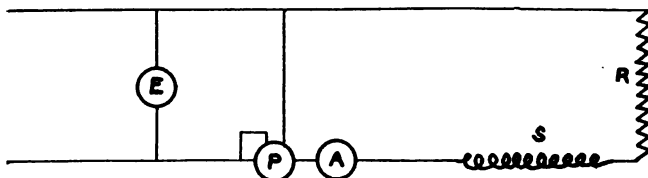
FIG. 41
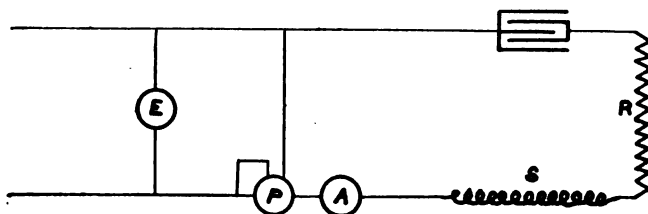
FIG. 42

If now we insert in the circuit a condenser with a variable number of plates, Fig. 42, we shall find the current C and the power P slightly increased.

By increasing the number or area of the plates in the condenser, the current and power can be increased to very nearly the values they had before.

CHAPTER VIII

Measuring Instruments—Principles of direct-current meters;
calculation of meter constants for use of shunts and multi-
pliers; wattmeters. Alternating-Current Meters — Dyna-
mometer and hot-wire type. Resistance Measurements—
Volt-ammeter method; Wheatstone's bridge. The Magneto.
Units of Electrical Measurement.

STUDENT'S GUIDE

In this chapter we study the instruments used to
measure voltage, current, resistance, and power.
Numerical examples are given to show the method
of constructing meters which will be able to measure
within predetermined limits. A table of electrical
units is also given.

MEASURING INSTRUMENTS

Reference has been made in preceding chapters to
various measuring instruments the characteristics of
which will be outlined here. Voltage, current, and
power are the three quantities which we should be
able to measure if we are to be certain that a given
installation is doing the work expected or of which it
is capable.

The direct-current voltmeter and ammeter are
interchangeable, provided shunts and multipliers are
furnished. The meter itself consists of a small moving
coil suspended, on spiral springs and jewel bearings,
in the air gap between the pole-faces of a strong

DIRECT
    CURRENT
        METERS

SHUNT
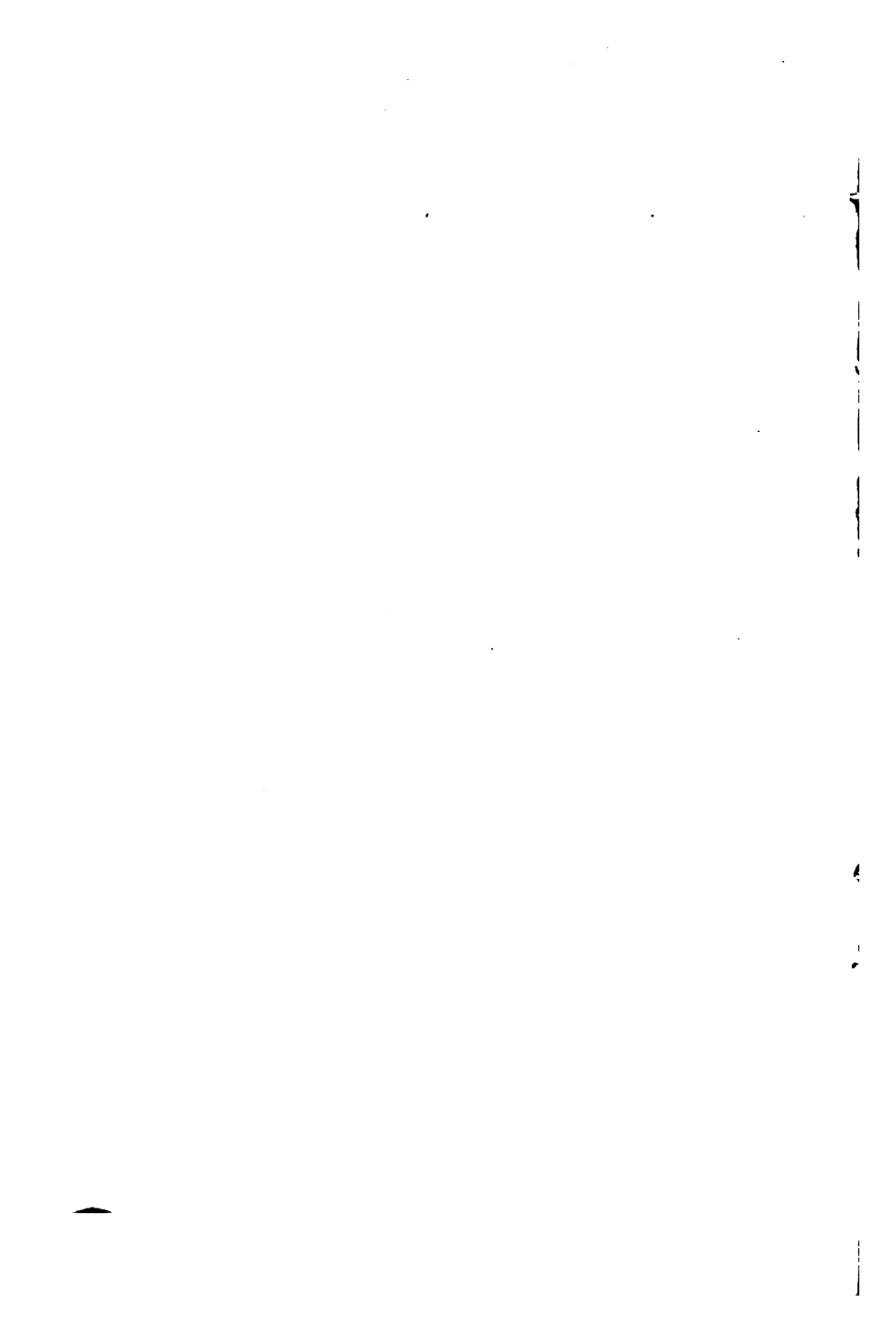
permanent magnet. The coil carries a pointer which moves over a scale when the coil rotates. Rotation of the coil is produced by motor action when a current passes through its turns, reacting with the field of the permanent magnet. The turning moment is proportional to the current in the coil, and rotation is restrained by the spiral springs with a force proportional to displacement; hence the pointer moves over a distance proportional to the strength of the current in the moving coil.

For very small currents the meter as described is satisfactory; but when larger currents are carried in and out of the coil, the heating effect on the springs which act as carriers for the current, causes errors, and the heating limit of the turns themselves is soon reached. Consequently shunts are always used, so that only a minute part of the total current passes through the coil. The scale of the meter is calibrated on the principle of the division of current; that is, the current carried by the shunt and the coil are in the inverse ratio of their respective resistances.

The same meter without the shunt is used as a voltmeter by inserting a large series resistance. Commercial meters are usually made so that full scale deflection is obtained with the passage of 0.01 ampere of current. Consequently if the resistance of the meter is known, the resistance which must be placed in series with it to limit the current to 0.01 ampere can easily be calculated.

For instance, let us calculate the resistance necessary in the shunt and "multiplier," as the series

resistance is called, for a meter which is to read a maximum of 50 amperes and 250 volts. Reasonable assumptions would be a meter resistance of 60 ohms, and a meter current of 0.01 ampere for full scale deflection.

To find the shunt resistance: the current carried by the shunt will be 49.99, and by the meter 0.01, when the pointer is at the top of the scale. The resistances must be inversely proportional to these values. Let R_s represent shunt resistance.

$$\frac{R_s}{R_M} = \frac{0.01}{49.99}$$

$$R_s = \frac{0.01}{49.99} \times 60 = 0.012 \text{ ohms}$$

The shunt we should expect, therefore, to be a short piece of metal of large cross-section, and this is in fact the form used. The value of resistance indicated as necessary is more easily arrived at by trial and comparison with standard instruments than by construction to exact dimensions, and that procedure is usually followed.

The calculation of the series resistance needed to construct a multiplier for the case assumed above is as follows: The voltage to be impressed when the full scale deflection is caused is 250; the current flowing must be 0.01 ampere. By Ohm's Law the total resistance $= R = \dfrac{E}{C} = \dfrac{250}{0.01} = 25,000$ ohms. The voltmeter itself has only 60 ohms in its coil, so that 25,000 — 60 = 24,940 ohms must be placed in the multiplier.

Multipliers are made by winding great lengths of fine wire on cards which are mounted in a ventilated box.

When a meter is wanted to measure power, one having the characteristics of both the ammeter and voltmeter is necessary, for the deflection of the pointer must be proportional to the product of volts and amperes.

To accomplish this the permanent magnet is replaced by a second, stationary coil of wire. If we cause a current to flow in the stationary coil proportional to the current in the circuit in which we desire to measure the power, and at the same time cause a current proportional to the voltage of that circuit to flow in the moving coil of our instrument, there will be produced a motor action as in the voltmeter and ammeter; but the deflection will be proportional to the product of two variables—that is, to the product of E and C, or proportional to power. Such an instrument is called a wattmeter. Current for the stationary coil is obtained as in the ammeter, and for the moving coil as in the voltmeter.

The ammeter and voltmeter described above can be used only on direct current, but the wattmeter will read alternating-current power, because the direction of the field caused by the stationary coil changes when the direction of the current in the moving coil changes, and consequently the rotating force is constant in direction; and while it varies with the growth and decrease of the current and voltage, the pointer will take up a position which indicates the average power.

Alternating-current ammeters and voltmeters are made with two coils in series, in order that the reversal of current may not reverse the force causing deflection of the meter pointer. If reference is made to the "left-hand rule" stated in Chapter V, it will be seen that if both flux and current direction are changed, the force exerted on the conductor will be unchanged. These are called dynamometer type instruments.

Alternating-current ammeters are also made on the principle of the heat developed in a wire carrying current, the wire expanding when heated, thus allowing a spring-controlled pointer to move over a scale calibrated to read amperes.

Illustrations shown are from the instrument catalog of the Wagner Electric Manufacturing Company.

In order to measure resistance, methods depending more or less directly on Ohm's Law are used. The voltmeter-ammeter method of resistance measurement applies the law directly by calculating resistance from simultaneous readings of current and voltage drop across the resistance in question.

Wheatstone's bridge is an apparatus much used for resistance measurements. It consists of means for balancing known resistances against the unknown which it is desired to measure. Kelvin's double bridge is a modification of the Wheatstone circuit by means of which very low resistances may be accurately measured.

The magneto is a small alternating-current generator with a permanent magnet field and two-pole revolving armature. The voltage generated by the ordinary

hand-driven testing magneto is comparatively high, but the current output is negligible.

Illustrations of meters and parts are from the instrument catalog of the Wagner Electric Manufacturing Company.

UNITS OF ELECTRICAL MEASUREMENT

The following list of electrical quantities and names of units is taken from the standardization rules of the American Institute of Electrical Engineers.

Capacity,	farad
Current,	ampere
Dielectric constant,	—(a number)
Electromotive force,	volt
Energy,	joule or watt-hour
Frequency,	cycle per second
Impedance,	ohm
Inductance,	henry
Magnetic flux,	maxwell
Permeability,	—
Phase,	degree
Power,	watt
Quantity of electricity,	coulomb
Reactance,	ohm
Resistance,	ohm

Since we have defined electricity as a means of transmitting energy, two of the terms in the list given above are obviously misnomers; they are "quantity" and "capacity." Quantity may be discarded entirely, while capacity will serve, failing to find a better word,

to describe the property of the condenser by virtue of which it neutralizes the effect of inductance. Capacity is calculated from the expression

$$\text{Capacity} = \frac{AK}{4D}$$

where A = area of active plates, D is thickness of dielectric, and K is a constant depending on the material of the dielectric. The rational unit becomes

$$\frac{\text{Area}}{\text{Length}} = \text{Length}$$

that is, inches or centimeters.

Current we have already defined as angular velocity.

Dielectric constant is a numeral.

Electromotive force or voltage has been assumed to be torque.

Energy may be expressed as horsepower hours.

Frequency is a number.

Impedance is the resultant resistance when a circuit has inductive characteristics.

Inductance is a magnetic quality.

Magnetic flux—imaginary lines indicating the direction of forces acting in the space about a current and in the neighborhood of magnets.

Permeability is a number.

Phase is the angular relation between various torques and rotating filaments.

Power is the rate of doing work and is expressed in horsepower.

Reactance is the component of impedance introduced by inductance.

Resistance is the counter effort made by various elements in an electric circuit to prevent rotation of molecular filaments.

The revised list then is:

Capacity,	a number
Current,	r.p.m.
Dielectric current,	a number
Electromotive force,	foot pounds of torque
Energy,	horsepower hours
Frequency,	cycles per second
Impedance,	a number
Inductance,	(magnetic)
Magnetic flux,	(magnetic)
Permeability,	(magnetic)
Phase,	degrees
Power,	horsepower
Reactance,	a number
Resistance,	a number

CHAPTER IX

General Remarks and Recapitulation.

If the analogy, as described in this work, were adopted as the basis for illustrating and explaining the various phenomena connected with the electric transmission of power, we believe it can be demonstrated that many of the difficulties which the student encounters would be removed. It does not seem to the writer possible for one to understand or explain how a small copper wire can convey a hundred or a thousand horsepower of energy by means of pulsations back and forth in or along or around the conductor. We have no experience with any such conditions producing similar results. The student may try to accept what he is told and often tries in vain. The analogy herein suggested deals with something familiar. The ordinary processes of reasoning lead to results which agree in nearly, if not all, cases with known and accepted facts, established by inductive study and experience. For the time at least, all reference to such terms as electric energy as related to transmission of energy by electricity could be dropped. The analogy does not call for any attempt to define electricity as something that exists, but only as a means of carrying energy. It translates the symbols of the basic law known as Ohm's Law into the common language of mechanics—namely, torque and angular velocity.

It does not limit the subject to any of these elementary ideas. It leaves plenty of room for imagination and investigation into ultimate realities and processes. But it leads to practical results by simple methods of reasoning. It brings ready insight into otherwise difficult problems. In a word, it permits the use of the deductive method of approach by quick and easy paths to a goal which it has taken many years to reach by the inductive method.

The foregoing is an outline of the ready application of our·analogy and its accompanying working hypothesis. It has shown, it seems to us, some remarkable results; results that are suggestive of the value and scope of the new approach to the study of electricity.

Some of these results are summarized as follows:

1.   The analogy leads to Ohm's Law by deduction.

2.   It translates the symbols and statements of that law into well understood mechanical terms— namely, torque, angular velocity, and energy or work.

3.   In connection with the working hypothesis it shows what may be the process of generating electricity in the dynamo and why the current is alternating, as generated.

4.   It reveals the use and operation of the commutator.

5.   It explains induction and shows that the induced current is 90° behind the primary current.

6.   It predicts and explains the transformer.

88 CHAPTER IX

7. It shows a ready application to the operation and effect of the condenser.

8. It shows how the effect of two out of phase torques, when combined, causes what is usually termed lag, and how to plot and measure the lag.

9. It indicates how an interrupted direct current in the primary coil of a transformer may give an alternating current in the secondary coil.

CHAPTER X

The Solution of Problems.

With a given circuit, certain results are invariably attained by successive applications of the same electromotive force, and by making use of certain rules and formulæ it is possible to predict what will happen in any given case. The following group of problems will illustrate some types of circuits, and the principles employed may be generally applied.

PROBLEM 1

To find the current when a known voltage is impressed on a combination of resistances.

Discussion. (See definition of electric circuit, Chapter III.)

Resistances are said to be in series or parallel according to whether the rotating filaments are all contained in one conductor or divided among two or more. When the whole current of a line is sustained at once in two or more resistances, those resistances are said to be in series; but when the filaments divide at one junction, part extending through one resistance and the rest through others, uniting at a second junction and forming one body thence through the line, the resistances are said to be in parallel.

Resistances in series are added to find the total or equivalent resistance. The equivalent or total

resistance of several resistances in parallel may be found as in Fig. 43.

In the circuit shown, current in branch a is, by Ohm's Law,

$$C = \frac{12}{2} = 6 \text{ amperes}$$

In branch b
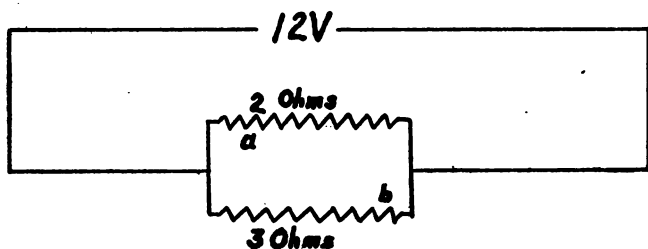
$$C = \frac{12}{3} = 4 \text{ amperes}$$

FIG. 43

The line current therefore is
$$I_L = 6 + 4 = 10 \text{ amperes}$$
The equivalent resistance $R\theta$ is that resistance with which a and b might be replaced without changing the value of I_L.

From Ohm's Law

$$R = \frac{E}{C}$$

$$R_L = \frac{12}{10} = 1.2 \text{ ohms}$$

We may also write

$$\frac{E}{R_a} = \frac{E}{R_a} + \frac{E}{R\theta}$$

or

$$\frac{1}{R} = \frac{1}{R_a} + \frac{1}{R_3}$$

and in general

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

Using the general equation, the student should check the value of R_L given above.

PROBLEM 2

A number of "exit" signs and fire-alarm boxes are to be illuminated with low-voltage lamps run from storage batteries, the purpose being to have a source of power entirely unaffected by any accident to the regular lighting circuits of the building.

The lamps available are 6-volt 2-ampere size, and the storage battery consists of a number of lead cells of 32 ampere-hour capacity. (See Chapter III.)

If 20 lamps are to be used 11 hours a day, how many storage cells will be needed and how should they be arranged, and how connected to the lamps?

Solution. The lamps should be connected in parallel in groups as indicated in Fig. 44. If placed in series a higher voltage would be required, and the burning out of one lamp would put all out of service. The groups are made as small as convenient to avoid the large line drop of voltage and consequent dimming

of lamps at the end of the line, which would be un-
avoidable if one long parallel row were used.

Three lead cells in series are known as a nominal
6-volt battery.

BATTERY

LAMPS

Fig. 44

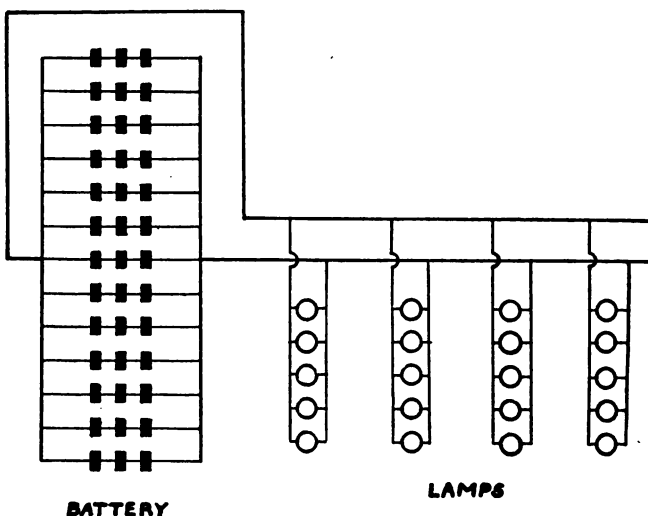Twenty of these lamps require 40 amperes.

From Chapter III we find that when a lead cell is
rated at 32 ampere hours, its normal current is 4
amperes; but it can give 4 amperes for but 8 hours,
and our problem requires an 11-hour run. We must
therefore find out what current each cell can give
when discharged at the 11-hour rate.

$$\frac{32 \text{ ampere hours}}{11 \text{ hours}} = 3 \text{ (approx.) amperes}$$

but we can get more than 32 ampere hours from a battery if it is discharged more slowly than at the 8-hour rate. Let us say 3.25 amperes is a safe value to assume. Then we are to have three cells in series and enough groups to make 40 amperes.

$$\frac{40}{3.25} = 12.3$$

Hence we shall need thirteen groups of cells in parallel, with three cells in series in each group.

It is to be noted that only in case the voltage of the circuit is that of a single cell will the sum of the ampere hours of the cells and the ampere hours of the load be the same. Ampere hours of cells in parallel are additive; in series the ampere hours are equal to those of any one. The energy expended, however, must not be confused with ampere hours; energy = ampere hours × volts.

These batteries would need to be charged every night.

PROBLEM 3

What is the cost of heating an electric soldering iron which is designed for a voltage of 110 and has a resistance of 35 ohms? Energy costs 5 cents per kilowatt hour. The iron is in use 8 hours a day.

Solution.

$$C = \frac{E}{R} = \frac{110}{35} = 3.14 \text{ amperes}$$

$$C^2R = (3.14)^2 \times 35 = 345 \text{ watts}$$

$$345 \times 8 \text{ hours} = 2,760 \text{ watt hours}$$

$$\frac{2,760}{1,000} \times 0.05 = \$0.138, \text{ cost per day}$$

PROBLEM 4

A portable electric drill has a $\frac{1}{4}$ horsepower motor operating at 110 volts. If the drill works at an average of five-eighths full load for 5 hours per day, when power costs 4 cents per kilowatt hour, how much does it cost to run the drill?

Solution.

$$\frac{1}{4} \text{ horsepower} = \frac{1}{4} \times 746 = 186.5 \text{ watts}$$
$$186.5 \times \frac{5}{8} = 116.5 \text{ watts}$$
$$116.5 \times 5 \text{ hours} = 582.5 \text{ watt hours}$$
$$\frac{582.5}{1,000} \times \$0.04 = \$0.0233$$

cost per day.

PROBLEM 5

A manufacturer is going to buy a 110-volt direct-current generator to supply the following: ten 1-horsepower motors; sixty 40-watt lamps; twenty 100-watt lamps; one 7.5-horsepower motor; two 5-horsepower motors; charging current for a 500 ampere-hour 24-volt storage battery.

What capacity generator would be required?
Solution.

$$10 \times 746 = 7,460 \text{ watts}$$
$$60 \times 40 = 2,400 \text{ watts}$$
$$20 \times 100 = 2,000 \text{ watts}$$
$$7.5 \times 746 = 5,600 \text{ watts}$$
$$2 \times 5 \times 746 = 7,460 \text{ watts}$$
$$\frac{500}{8} = 62.5 \text{ amperes}$$

normal charging current for the battery.

$7,460 + 2,400 + 2,000 + 5,600 + 7,460 = 24,920$ watts

$$\frac{24,920}{110} = 226.54 \text{ amperes}$$

motor current.

$$226.54 + 62.5 = 289 \text{ amperes}$$

It is not likely that the whole load would be on at any one time; consequently we must assume some load factor, to indicate the proportion of the total connected load which may be expected to be carried at any time. Of course, if the total load might possibly be thrown on at once, no reduction from the 289 amperes could be taken.

Assuming a load factor of 85 percent, the maximum load will be

$$289 \times 0.85 = 246 \text{ amperes}$$

$$246 \times 110 = 27,000 \text{ watts} = 27 \text{ kilowatts}$$

capacity of generator required.

PROBLEM 6

A starting-box for a 5-horsepower 220-volt shunt motor is to be remodeled for use with a 7.5-horsepower motor of the same voltage. What resistance will be necessary in the armature circuit?

Solution. A 7.5-horsepower motor takes

$$\frac{7.5 \times 746}{220} = \frac{\text{watts}}{\text{volts}} = 25.4 \text{ amperes}$$

at full load. Shunt motor starting-boxes are designed to allow about 150 percent of full-load current to flow in starting. At start, the motor has no counter

electromotive force (see Chapter IV), and so the resistance of the box must limit the current to

$$25.4 \times 1.5 = 38.1 \text{ amperes}$$

Hence

$$R = \frac{220}{38.1} = 5.77 \text{ ohms}$$

The resistance of the armature, which would be small, has been neglected.

There is another point to be considered before undertaking the actual remodeling of the box. A calculation smilar to the above gives the resistance of the box as it stands as about 8.65 ohms, with a momentary current at starting of 25.44 amperes. If the resistance coils already in the box were merely cut shorter, so that the total resistance came down to the required 5.77 ohms, the larger current might burn them out. Consequently new coils should be supplied having larger current capacity than those originally in the box.

Problem 7

The phase voltage of an alternator is 127; the windings are connected in Y (or star). A three-phase induction motor is running on current supplied by this generator, its input being 3 kilowatts and its power factor 85 percent. What current is flowing in the line? (See Chapter VI for definitions and diagrams.)

Solution. Using the notation of Chapter VI, we have, in the star system,

$$E_L = \sqrt{3}\, E\, \emptyset$$

The line voltage then equals

$$E_L = \sqrt{3}\; 127 = 220$$

Three-phase power (in either system) is there stated as

$$P = \sqrt{3}\, E_L\, I_L \cos \emptyset$$

We have given

$$P = 3 \text{ kilowatts} = 3,000 \text{ watts}$$
$$E_L = 220 \text{ volts}$$
$$\text{Cos } \theta = 0.85$$
$$\therefore P = \sqrt{3}\; 220\; I_L\; 0.85 = 3,000 \text{ watts}$$

$$I_L = \frac{3,000}{220 \times \sqrt{3} \times 0.85} = 9.27 \text{ amperes}$$

PROBLEM 8

The installation of problem 5 is to be changed over to alternating current. If the power company supplies 2,200-volt 60-cycle current at 3½ cents per kilowatt hour, what will the motor equipment consist of, and how much will the power cost per working day of 8 hours, with one battery charge every third day?

Solution. From a survey of the list of equipment it seems likely that at least five 1-horsepower motors and one of the 5-horsepower motors would require variable speed. Hence direct current must be supplied, since there is no good variable-speed alternating-current motor. Direct current must also be supplied for charging the storage battery. This would probably be done at night. The most economical way to charge the battery would be by installing a

low-voltage, motor-driven generator; but since the charge is given but once in three days and then at a time when the direct-current generator is supplying no other current, it may be run at reduced voltage, thus wasting less energy in regulating resistance.

Five 1-horsepower 110-volt motors (direct current) require

$$\frac{5 \times 746}{110} = 33.9 \text{ amperes}$$

One 5-horsepower 110-volt motor (direct current) requires the same current. The total, $2 \times 33.9 = 67.8$ amperes, is larger than that required for charging the storage battery; hence the motors are the determining factor.

$$67.8 \times 110 = 7,458 \text{ watts}$$

Thus if we are buying "close," we shall get a 10-horsepower induction motor, 220 volts, driving a 7.5-kilowatt, 110-volt, direct-current generator.

The lamps in the plant may be run from a middle tap on the secondary of one of the transformers, half the lighting load between the middle and one line, and half between the middle and the other line of that phase.

That leaves five 1-horsepower motors, one of 5 horsepower, and one 7.5-horsepower motor, all 220-volt induction type.

$$60 \times 40 = 2,400 \text{ watts}$$
$$20 \times 100 = 2,000 \text{ watts (lamps)}$$

$$\frac{4,400}{220} = 20 \text{ amperes (current for lamps)}$$

Five 1-horsepower motors take 16.95 amperes (on unity power factor).

One 5-horsepower motor takes 16.95 amperes.

One 7.5-horsepower motor takes 25.43 amperes.

One 10-horsepower motor takes 33.9 amperes.

The total alternating-current motor current is 16.95 + 16.95 + 25.43 + 33.9 = 93.23 amperes at unity power factor. Assuming that the average power factor of the motor load is 80 percent, the volt-ampere capacity of the transformers must be

$$V \left(\underset{\text{motors}}{\frac{93.23}{0.8}} + \underset{\text{lighting}}{20}\right) = 136.54 \text{ amperes} \times 220 \text{ volts} = 30$$

kilovolt amperes.

If the shop load factor is 75 percent,

$$0.75 = 93.23 \times 220 \times 8 \times \frac{1}{1,000} = \text{kilowatt hours}$$

motor input per day = 123.06.

Battery charging (total) 10-horsepower motor run at average of half load for 10 hours = 5 × 746 × 10 = 37.3 kilowatt hours; per day = 12.43 kilowatt hours.

Lamps, 4.4 kilowatts × 8 = 35.2 kilowatt hours.

$$(\underset{\text{motors}}{123.06} + \underset{\text{lamps}}{35.2} + \underset{\text{battery}}{12.43}) \, 0.035 = \$5.97 \text{ per day}$$

PROBLEM 9

A certain three-phase 60-cycle transmission line operating at 100,000 volts has the following line con-stants: length, 25 miles; inductance, 0.119; capacity, 0.396 (microfarads); resistance, 40 ohms. What

current will flow at the power house when there is no load at any point in the line?

▎ *Solution.* The accurate solution of such problems is difficult; an approximation may be obtained by the simple method used below.

$$C = \frac{E}{\sqrt{R^2 + X^2}}$$

$E = 100,000;$

$R = 40;$

$$X = LW - \frac{1}{CW} = 0.119 \times 2\pi 60 - \frac{1}{0.396 \times 10^{-6} \times 2\pi 60}$$

$W = 2\pi f$

$$X = 0.0067 \times 10^6$$

R is so small compared with X that

$$\sqrt{R^2 + X^2} = 0.0067 \times 10^6$$

very closely.

$$C = \frac{10^5}{0.0067 \times 10^6} = 14.93 \text{ amperes}$$

APPENDIX

Properties of the Sine-Curve. Distorted Waves of Current and Voltage—Form factor; peak factor. Mean and Effective Values of Sine-Curve—Discussion; calculation. Torque and Speed of Rotation of Molecular Fibers.

PROPERTIES OF THE SINE-CURVE

When a coil of wire is revolved in a magnetic field as shown in Fig. 45, the velocity with which the coil cuts the lines of force is found by resolving the uniform velocity of the coil in its circular path into two rectangular components, one component parallel to the lines of force. This component has no effect on the coil. The component at right angles to the lines of force is the rate at which the coil cuts the lines of force. The e.m.f. and current generated in the coil at any instant are proportional to this velocity rate. This rate of cutting the lines of force is $V \sin \theta$, in which V is the velocity of the cutting wires and θ is the angle which the plane of the coil makes at any instant with the horizontal.

A curve drawn as in Fig. 45 is called a sine-curve, and for the reasons above stated (see Chapter IV) such a curve is used to represent either the e.m.f. or the current in an alternating-current circuit.

Let OA be such a curve. Let it be required to find the mean ordinate of one loop of the curve; that is, the ordinate which multiplied by OA will give the

area of the rectangle $OABC$. If R is the radius of the circle, the equation of the curve is $y = R \sin \theta$, in which θ is the variable angle R makes with OX. Then representing the mean ordinate by E_M, we have
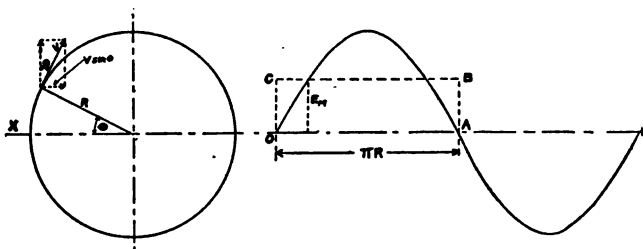

FIG. 45

$E_M \times \pi R$ = area of rectangle = area of sine-curve =

$$= \int_0^\pi y dx$$

But

$$dx = R d\theta$$

$$\therefore E_M \times \pi R = R^2 \int_0^\pi \sin \theta d\theta$$

$$= R^2 \left(-\cos \theta\right)_0^\pi = 2R^2$$

$$E_M = \frac{2R}{\pi}$$

Thus the area of a sine-curve is proportional to the square of its maximum ordinate.

WAVE DISTORTION

In Chapter IV, reference was made to variation of wave form in alternating current and voltage. To give comparative values to such distortions, the terms *form factor* and *peak factor* have been adopted. The

ratio of effective to average value is called form factor; and the ratio of maximum to effective value iscalled the peak factor. For the sine-curve the values are:

$$\left.\begin{array}{l} \text{Form factor } = \dfrac{\pi}{2\sqrt{2}} = 1.11 \\[2mm] \text{Crest factor } = \sqrt{2} = 1.414 \end{array}\right\} \text{sine-curve only}$$

It is to be noted that a sharply peaked wave will be likely to have a form factor greater than 1.11, and a flat-topped wave will in general have a lower form factor than a sine-wave of equal effective value.

MEAN AND EFFECTIVE VALUES—THE POWER CURVE

The following is introduced to clear up in the student's mind any confusion into which he may have fallen concerning the use of mean and effective values of current and voltage.

Figure 46 is a reproduction of two curves, C and L, from Fig. 22, with the addition of the power curve P, which is obtained by multiplying each ordinate of the voltage curve L by the corresponding ordinate of the current curve C.

If the base be properly taken, the area under curve P will represent the energy expended during the half cycle. (See curves, Figs. 38 and 39.)

When the mean ordinates of curves C and L are known, the energy can be obtained from the area of L as follows:

Power = effective volts $\times$ effective amperes

$$\frac{\text{effective volts}}{\text{mean volts}} = 1.11; \qquad \frac{\text{effective amperes}}{\text{mean amperes}} = 1.11$$

Hence

Power = mean volts $\times$ mean amperes $\times$ $(1.11)^2$
and the energy per half cycle, *when the mean current
is unity*, is

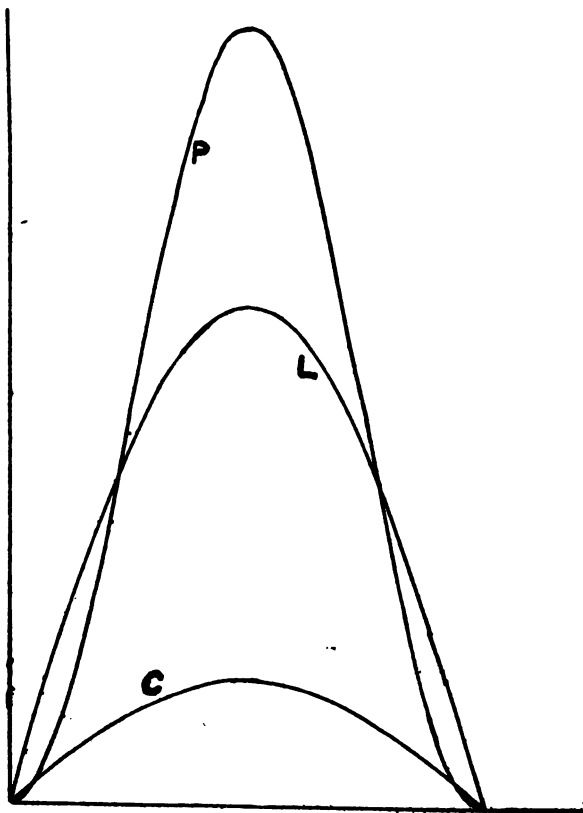Energy = area of voltage curve $\times$ 1.2321

FIG. 46

CALCULATION OF MEAN AND EFFECTIVE VALUES

Distinction between mean and effective values of sine-wave functions: Alternating-current machinery is designed to operate as nearly as possible on pure sine-wave forms of voltage, current, and magnetic flux variations, calculations and predeterminations being thereby made more easily.

It should be noted that voltmeters and ammeters do not read the geometric mean of voltage or current, but a value slightly higher, called the "effective" value.

If $i = \sin \theta$ is the equation of an alternating current, i^2R is the instantaneous power, and substituting $I^2 \sin^2 \theta$ for i^2, we have

$$I^2R = I^2 \sin^2 \theta R$$

Let us find an expression for the average power, which can be done by integrating the expression $\dfrac{I^2\sin^2\theta R}{\pi}$ between the limits of O and π; that is, over one loop of the curve of $i = I \sin \theta$

$$P = \int_o^\pi \frac{I^2 sin^2\theta R}{\pi} \, d\theta = \frac{I^2R}{\pi} \int_o^\pi sin^2 \, \theta d\theta =$$

$$\frac{I_2R}{\pi} \int_o^\pi \left(\frac{1}{2} - \frac{1}{2}\cos 2\theta \right) d\theta = \frac{I^2R}{2\pi} \int_o^\pi (1 - cos \, 2\theta)d\theta$$

$$= \frac{I^2R}{2\pi} \left(\int_o^\pi d\theta - \frac{1}{2}\int_o^\pi cos \, 2\theta \, 2d\theta \right) = \frac{I^2R}{2\pi} \left[\, \theta \, \right]_o^\pi - \frac{1}{2} \, sin$$

$$2\theta \left. \right]_o^\pi = \frac{I^2R}{2\pi} \left(\pi - \left(\frac{0}{2} - \frac{0}{2} \right) \right. = \frac{I^2R}{2}$$

Now from this expression for average power we can obtain the value of alternating current which is the equivalent of some direct current which would give equal power, and this value of alternating current is called the effective or virtual value of the current, and is the value read by alternating-current meters.

$$\text{Average power} = \frac{I^2 R}{2} = \left(\frac{I}{\sqrt{2}}\right)^2 R$$

whence it is seen that $\frac{I}{\sqrt{2}}$ is the value which corresponds to the direct current which will have the same heating effect and provide the same power as the alternating current which has I for its maximum. When an engineer refers to an alternating current of 10 amperes, he refers to this meter value or effective value.

Whence the effective value and maximum values are related thus:

$$I_{ef.} = \frac{I_{max.}}{\sqrt{2}} = 0.707 \ I_{max.}$$

The effective current value is slightly higher than the average ordinate of the corresponding sine-curve, as will be seen from the following integration:

$$i_{ave.} = \int_{0}^{\pi} I \ sin \ \theta d\theta \times \frac{1}{\pi} = \frac{I}{\pi} \int_{0}^{\pi} sin \ \theta d\theta$$

$$= \frac{I}{\pi} \ (- \ cos \ \theta)_0^{\pi} = \frac{I}{\pi}\left[- (-1-1)\right] = \frac{2I}{\pi}$$

Whence

$$i_{ave.} = \frac{2}{\pi} \times I_{max.} = 0.636 \ I_{max.}$$

Whence it appears that the average current is 0.636 of the maximum, while the effective is 0.707 times the maximum.

TORQUE AND SPEED OF ROTATION OF MOLECULAR FIBERS

While it-serves no immediately useful purpose, it may be interesting to examine the equation $P = M \times A$ in an attempt to assign possible values to the torque and angular velocity of individual molecular fibers.

The diameter of the elementary fiber can hardly be less than that of a single molecule, though it may be greater. From the table of molecular dimensions given by the Smithsonian Physical Tables, it may be assumed that 4×10^{-8cm} ($= 1.575 \times 10^{-8}$ inches $= 0.00000001575$ inches) is an index of the order of magnitude of the copper molecule.

Intermolecular interstices seem always to be characteristic of matter. If we assume that 80 percent of the apparent area of a certain wire is composed of cross-sections of molecular fibers, there will be in the wire a total of F fibers, where

(1) $$F = \frac{0.8S}{s}$$

S = area of wire, and s = area of filament.

For purposes of calculation let us use a copper wire having a diameter of 0.5 inch, and let us assume that it is a part of a line which is transmitting 800 horsepower in a 1,200-volt system. This would correspond to a feeder wire of an interurban electric railway.

The area of the wire is

(2) $S = \pi R^2 = \pi(0.25)^2 = 0.1962$ square inch

The area of a filament is

(3) $s = \pi r^2 = \pi\left(\dfrac{1.575 \times 10^{-8}}{2}\right)^2 = \pi \times 0.62 \times 10^{-16}$

$= 1.947 \times 10^{-16}$ square inches

Hence the number of molecular fibers is

(4) $F = \dfrac{0.8 \times 0.1962}{1.947 \times 10^{-16}} = 0.0807 \times 10^{16}$

$= 807,000,000,000,000$

The assumed rate of transmission was 800 horse-power, or

$800 \times 33,000 = 26,400,000$ foot pounds per minute

Therefore the rate per fiber is

(5) $\dfrac{26,400,000}{807,000,000,000,000} = \dfrac{264 \times 10^5}{807 \times 10^{12}}$

$= 0.3272 \times 10^7$

$= 0.00000003272$ foot pounds per minute per fiber

This represents P in the equation

(6) $P = MA$

whence

(7) $P = 0.3272 \times 10^7 =$ torque per fiber $\times 2\pi N$

It will be seen that an arbitrary value must now be assigned to torque or revolutions, and the difficulty is presented of attempting to find a reason for making whatever assumption may be decided upon. For it is evident that if the problem be set before ten individuals, there might be presented ten different solutions from which it would be impossible to choose one as being better than the others. This fact is one

more point of evidence in support of the rotational theory of electrical transmission.

The current flowing in this wire may be found from the conditions previously stated. Eight hundred horsepower is being transmitted at 1,200 volts. There are 746 watts in one horsepower, hence

$$W = 800 \times 746 = 596,800 \text{ watts}$$

(8) $\qquad C = \dfrac{W}{E} = \dfrac{596,800}{1,200} = 497.33 \text{ amperes}$

If we examine Equation 6, we find that the right-hand member consists of two quantities which may be expressed thus:

M = total torque per fiber = torque per volt $\times$ number of volts;

$A = 2\pi N$ = angular velocity, where N = r.p.m. per ampere $\times$ number of amperes.

In the present example

(9) $P = 0.3272 \times 10^{-7}$ foot pounds per minute (from Equation 5)

(10) $\qquad\qquad M = 1,200\ T$

where

1,200 = number of volts;

T = torque per volt, per fiber.

(11) $\qquad A = 2\pi N = 2\pi(497.33n)$

where

497.33 = number of amperes;

n = r.p.m. per ampere.

Let us assume for purposes of calculation that $n = 10^4 = 10,000$. A, then, $= 2\pi(497.33 \times 10,000) = 31,200,000$; or $A = 31.2 \times 10^6$.